Dynamic Parking Pricing

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24 October 2019

Submitted in partial fulfillment of a structured masters degree at AIMS South Africa

Abstract

Over the years there have been numerous solutions addressing the major issues of cruising cars looking for parking, which causes an effect on the environment and people's health. The *dynamic parking pricing* thesis is whereby solutions are solved for such a problem, in a business-wise perspective which will illustrate how suppliers would be able to benefit within a competitive market. This can be done by looking at the relationship between the quantity and price of the parking. In this thesis, there will be a development of a price model function based on the demand and the available space, whereby the function which optimizes the profit of a parking owner will be created using a parking pricing model function. The important part which will be done well is to optimize the profit using the (1+1)-evolutionary strategy within the bi-level algorithm.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Mzwakhe Mandlakhe Mthethwa, 24 October 2019

Contents

1. Introduction

1.1 Background

There has been a report which estimates that in major European cities and Boston cities, there are over half of the total amount of cars which have been cruising for parking over time and is said to be one of the major problems (Anderson et al., 2004).The number of parking traffic has increased in city centrals in such a manner that prominent economists have tried to come up with efficient methods to resolve the matter through road pricing. The road pricing helps in the absenter of parking pricing and in which an optimal policy is needed for the method to be successfully implemented and operated.

The implementation of on street parking was first introduced in 1935 by the city of Oklahoma by using parking meters for parking spaces (Mechanics, 1935). In the recent years, rates have been implemented on metered off street parking and at most times this becomes cumbersome towards citizens pockets compared to free on street parking and therefore resulting in a revival of traffic and cruising once again.

The search of available parking by shoppers or drivers in a city has caused a lot of worrying ever since the invention of the automobile due to increasing populations and little parking space. A research done in 1927 experienced that the amount of cruising congestion by cars searching for available parking was estimated to be between 30% and 50%(Alternatives, 2008).The off street spaces and municipal garages were one of the cities developments in the 1960s in order to try and avoid the problem of traffic and cruising of automobiles because of the shortage of parking areas, which did have a positive impact in slow density areas, new development areas as well as suburbs but it did not show great impact in urban areas where there is a high density population with the competition being remarkably still high with off street parking being present.

During the 1970s to 1980s, the increase in lack of responsibility by politicians led to an increase in cruising parking once again in many cities which forced automobile drivers and shoppers to occupy parking spaces made for public transportation and delivery vehicles, while studies also illustrate that traffic congestions also results in one third of the traffic jams encountered due to cars in search of a place to park on a daily basis and due to this matter there have been reports in the USA with increasing traffic congestion, it has been the main contributor to air pollutions, drawing in numerous disease to humans and nature(Tilahun and Di Marzo Serugendo, 2017). The recent development of fluctuations in off road parking prices is also known to contribute to the problem.

1.2 Objectives

In this research thesis the aim is to derive a pricing parking model of any parking owner, and it should depend on the available space with the predicted demand of cars which need parking. It also looking on how can that pricing parking model affect the profit of the parking owner. The bi-level optimization model should show the interaction between the pricing decisions of the two competitors which will affect the profit of each other. Hence, the main objectives of this study are:

1. To construct a dynamic pricing model based on the dynamic change in demand and the limited resource available. This creation of a model function is done by using the demand equation in the way which is shown by Zeder (2019), with a definition about the demand proposed by Chappelow

(2019).

2. The second objective is to create a profit function that will depend on the price model which will be derived from the first objective.

3. Considering two owners with two different properties who are business competitors, the objective is to find the general way to get the demand of profit.

4. The last one is to use the (1+1)evolution strategy under metaheuristic algorithm to solve the bi-level optimization of two profit functions, to show the result of how the two owners decides regarding maximizing their profit into the plots.

1.3 Layout the thesis

In this thesis we have the following outline:

Chapter 1: Introduction, and this is where we have the following:

Background, it where there is a root of a problem which is being solved and discussed by going through its history. *Objectives*, give us the aim of a thesis and also the four steps needed to be done to achieve our aim. In *Layout of this thesis*, it where we get the order of this thesis, which illustrates where everything is found. *Contribution* which illustrates where this thesis will help or what input it would contribute to In the department of mathematical Sciences and any other study.

Chapter 2: Dynamic parking pricing and profit function, we have the following,

In *the pricing parking model*, a brief background is provided which shows how to develop the model function, and it features *the plot and variables with it's description*, *the model function* and the example. *Profit function* is where a price model is been used to develop a profit function from a general profit formula.

Chapter 3: Multiple parking place, we have the following:

In *competing parking companies*, there is a brief reasoning for the solution to address a problem of two business competitors and their demand function in terms of price, and it features the *demand modelling* and *the profit function for two parking*. In *demand as a function of prices*, explains and shows how to get the demand function of both parking profits using three cases in each. While in *bi-level formulation of pricing problem*, there is a clarity of Bi-level optimization formulation and how it is connected to the (1+1) evolutionary strategy.

Chapter 4: Numerical Results, we have the following:

In *Set-up* which features the *software* where the name and a version of programming language is being described, and where a link to the tool is provided for obtaining the code online, and the *algorithm parameter setting* which have brief explanation of how does the code work with some parameters being set. In *data*, there is data in a form of paragraphs explaining some values assumed for some variables and illustrated in a tabular format. In *results*, will show the two different plots of results after using Matlab. It shows how can the two parking owners decide to modify their price in order to optimize their profit even if competition exists.

Conclusion gives the take base on the aim and the objective of the thesis, if there were accomplished or not.

1.4 Contribution

The study will propose a dynamic way of pricing parking depending on the predicted demand towards gaining a maximum profit. A bi-level model will also be used to propose on how multiple decision makers with their own parking places interact with each other. Hence, it helps the decision maker in the industry to make an informed decision towards optimizing their profit.

2. Dynamic parking pricing and profit function

2.1 The pricing parking model

Parking price setting depends on a number of issues including the available parking slots and the demand for a parking slots. These again changes over time and varies on different days. Considering a particular day where the price needs to be set, the time frame can be divided into slots (see 2.1) where a price can be used for the given period of time and updated after a fixed time width, say δ.

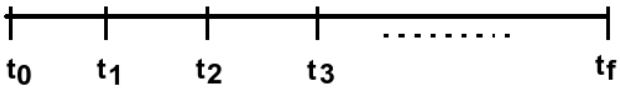

Figure 2.1: The demand linear plot

$$\delta = t_{i+1} - t_i \tag{2.1.1}$$

The price-quantity relation has been studied intensively by different economists. It has an inverse relation where the price reduces when there are more quantity and vice versa (Greenlaw and Shapiro, 2019).

2.1.1 Price-Quantity Relation.

The price quantity relation has an inverse relation where the quantity increment results the decrement of the price. Different curves can be used to represent this relation. Linear model is mostly used, and hence, this research considers a liner relation between price and quantity. It should be also highlighted the choice of the curve doesn't affect the analysis presented in this report, rather a similar argument and extensions can be done on other curves as well.

The price is also directly proportional to the demand implying that with fixed quantity the demand increase will lead to the increase with the price. In a price - quantity model the increase in the demand will move the price curve upward and decreasing in the demand moves the curve downward (see Fig. 2.2).

The assumptions in this project are: there is a minimum and maximum possible parking prices, μ and P_m, respectively. Let the maximum number of parking slots in a parking place be N, where the minimum is set to be q_0 which usually is 1.

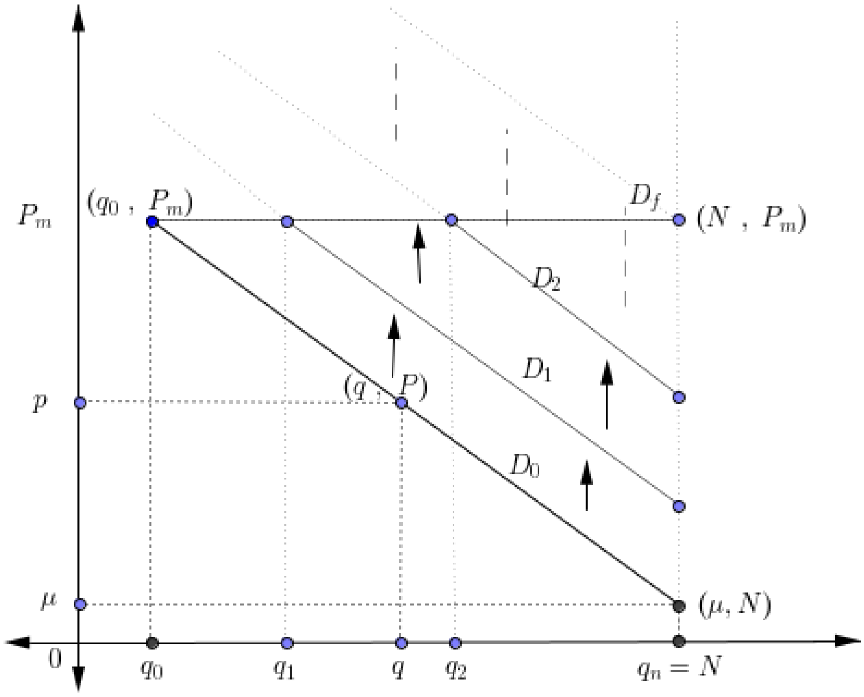

Figure 2.2: The demand linear plot

$$\text{for} \quad D_i \quad \text{where} \qquad P = \begin{cases} mq + b & \text{if} \quad q \in [q_i, N] \\ P_m & \text{if} \quad q \in [q_0, q_i] \end{cases} \tag{2.1.2}$$

The demand curves in the above figure 2.2 is taken as $D_0 < D_1 < \cdots < D_f$, and our demand will be $D = (D_0, D_1, \ldots, D_f)$

In below we have variables of a model from the graph and their description:

P $\rightarrow$ for price function which will be a subject of the formula.

μ $\rightarrow$ Proposed minimum price for period of α time in minutes.

P_m $\rightarrow$ Proposed maximum price.

N $\rightarrow$ is the total number of parking

q $\rightarrow$ number of the available parking space.

q_0 $\rightarrow$ minimum number of the available parking space.

The above plot of figure 2.2 is taken from the fact that if we have the parking owner who wants to develop a model which gives the initial parking price each independently at a certain time, depending on the number of available parking and the demand at that time. To develop this model function we are assuming that the parking owner will need to give the price range which will be the boundary amount where this pricing model won't go above and below the maximum price P_m and minimum price μ of this range respective.

When there are maximum amount of the available parking with no demand, the first car to park will be charged the minimum price which is μ. But if the demand which is the number of cars that need a space to park increase, it will affect the initial price to be not at the minimum but higher even if the parking were not occupied.

2.1.2 The model function.

By using an concept from economic, we will have the inversely demand equation which is:

$$P = mq + b \tag{2.1.3}$$

when there is no demand or having zero demand. We will now use the two coordinates (q_1, P_m) and (N, μ) from Fig 2.2 and substitute it into the equation 2.1.3 to find the gradient, which will be as follows:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{P_m - \mu}{q_0 - N} \end{aligned}$$

Substituting the gradient into the demand equation, we will have the following:

$$P = \left(\frac{P_m - \mu}{q_0 - N}\right) q + b \tag{2.1.4}$$

To find the value of *b* from the demand equation, we used one of the two coordinates that is being used to find the gradient above, if we choose it to be (N, μ) and substitute it into *q* and *P* of the equation 2.1.4. We will then have the following:

$$\mu = \left(\frac{P_m - \mu}{q_0 - N}\right) N + b$$

$$b = \mu - \left(\frac{P_m - \mu}{q_0 - N}\right) N \tag{2.1.5}$$

then by substitution Eq. 2.1.5 into Eq. 2.1.4, we will get:

$$P = \left(\frac{P_m - \mu}{q_0 - N}\right) q + \mu - \left(\frac{P_m - \mu}{q_0 - N}\right) N$$

Thus price parking model function that we are looking for is the following:

$$P = (q - N)\left(\frac{P_m - \mu}{q_0 - N}\right) + \mu \tag{2.1.6}$$

Finding a price in terms of demand, we consider that when the demand increases then a demand curve in figure 2.2 will shift upward, therefore our price function in Eq. 2.1.6 will yield to the following function:

$$\text{thus} \quad \boxed{P(q, D) = (q - N)\left(\frac{P_m - \mu}{q_0 - N}\right) + \mu + \gamma D} \qquad \text{where} \quad D \leq D_f \tag{2.1.7}$$

And the D_f will be a maximum or final demand where by it minimum price is corresponding with a maximum price P_m of the first curve.

From the general equation of the inversely demand in equation 2.1.3, we can consider the increase in demand of people who are looking for parking, which will yield into the following equation:

$$P = mq + b + \gamma D \tag{2.1.8}$$

The function of the final demand D_f can be derived by knowing the gradient of the equation 2.1.8 since it's curve is parallel to the curve of equation 2.1.7, we also know one coordinate of D_f as being (N, P_m).

The equation function of the last demand we are taking will be:

$$P = mq + b + \gamma D$$

since D_f is in point (N, P_m) we will substitute this coordinate in equation 2.1.7 above, and it will yield into the following:

$$\gamma = \frac{P_m - \mu}{D_f} \tag{2.1.9}$$

After substituting the γ which is in equation 2.1.9 into our price model in equation 2.1.7, our model will be:

$$\text{thus} \quad \boxed{P(q, D) = (q - N)\left(\frac{P_m - \mu}{q_0 - N}\right) + \mu + \left(\frac{P_m - \mu}{D_f}\right)D} \qquad \text{where} \quad D \leq D_f \tag{2.1.10}$$

In other words, for the initial price of an individual parking not to exceed the maximum price which is the upper boundary, we will have our price as

$$\begin{aligned} P &= \ min\ \{P(q, D),\ P_m\} \\ &= \ min\ \left\{(q - N)\left(\frac{P_m - \mu}{q_0 - N}\right) + \mu + \left(\frac{P_m - \mu}{D_f}\right)D,\ P_m\right\} \end{aligned}$$

.

$$P = \ min \left\{(q - N)\left(\frac{P_m - \mu}{q_0 - N}\right) + \mu + \left(\frac{P_m - \mu}{D_f}\right) D,\ P_m\right\} \tag{2.1.11}$$

Example

To check the different parking prices using Eq. 2.1.11, we will take variable demand D which will be incrementing with 10 unit, final demand D_f, initial number of available parking q_0, a total number of parking (N) and maximum price value P_m to be the known value. The number of available space (q) will varies in a function, and this will be as follows example:

- $$N = 6, \quad \mu = R10, \quad P_m = R50, \quad q_0 = 1, \quad D_f = 15$$
  $$P(q, D) = \frac{8}{3}D - 8q + 48$$
  $$\therefore P = \min \left\{\frac{8}{3}D - 8q + 48,\ 50\right\}$$

For the demand values from 1 to 100 with 10 increment is plotted below:

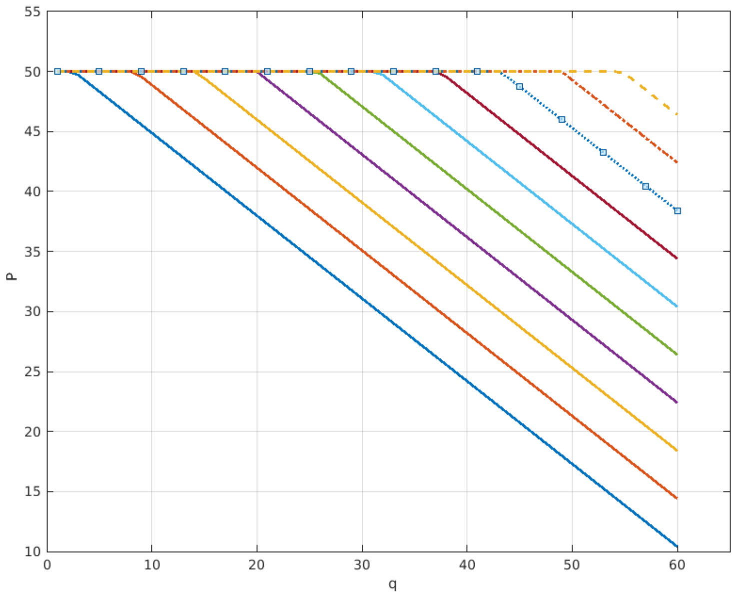

Figure 2.3: The demand linear plot

2.2 Profit function

As a business the parking place management wants to maximize their profit. This depends on how many slots are being used by drivers, as many drivers park in that parking area. The profit then will be the income, which is the unit price for a slot multiplied by the taken slots (slots in use) minus the total cost. At a particular time slot the income is given by:

$$I = P(N - q) \tag{2.2.1}$$

where P is the price which is multiplied by the used slots.

The cost on the other hand for parking places are not much changing. The salary, the cleaning and related are pre-known costs. With the assumption that no other costing exist, the total cost can be represented by C.

Hence, the profit function for a parking place is given by:

$$\mathcal{P} = P(N - q) - C \tag{2.2.2}$$

Using the price function given in the previous subsection, the profit can be given by:

$$\mathcal{P} = \left[min\left\{ \frac{P_m - \mu}{q_0 - N} + \frac{P_m - \mu}{D_f} D + \mu,\ P_m \right\} \right] (N - q) - C \tag{2.2.3}$$

It should be highlighted that, if a car is parked even for a second beyond δ unit of time it will be charged the next set price for the next δ period of time.

3. Multiple parking place

3.1 Competing parking companies

The parking industry, like any other business, is not a stand alone business. There is a competition between different parking places in a region to attract more customers and maximize their gain or profit. Each parking place management will set-up the parking slot price in a given period of δ time with the objective of keeping its customers from going to other parking places and attracting customers from other parking places. In order to do so, the demand can be predicted ahead (for example based on multi-agent system coupled with Markov chain approach Tilahun and Di Marzo Serugendo (2017)). The price then can be expressed as a function of demand which later can be used to optimize profit as a function of the price. Demand modelling will be discussed in detail, in the following subsection. In this report two competing parking places are considered.

3.1.1 Demand modelling.

Consider two parking places, parking-1 and parking-2, competing each other in an area. Look at a possible hypothetical map for the parking places as well as the buildings people may go after parking.

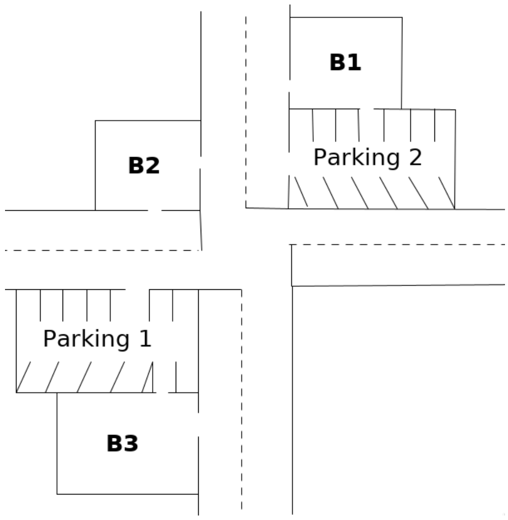

Figure 3.1: Hypothetical example of two competing parking places

Based on the figure above, people going to building 1 (B1) will prefer parking-1 as far as the price is acceptable. Acceptable price means $P_1 \leq P_2 + \overline{P_{11}}$, where $\overline{P_{11}}$ refers to the amount of money they are willing to pay more to stay in parking-1.

People interested to go to building 2 (B2) preferes to park in parking-2. However, they may park in parking-1, if it is cheaper than the price in parking-2. It cheap if $P_1 \leq P_2 - \overline{P_{21}}$, where $\overline{P_{21}}$ refers to the amount in which people who prefer parking-2 will migrate to parking-1 if the price is at least less than by that amount.

People interested going to building 3 (B3) are interested in the cheapest parking.

Hence, there are three types of demand (customers) a parking management needs to consider before setting the price.

The additionally introduced variables and parameters are:

N_i $\rightarrow$ the total number of parking place for the i^{th} parking place.

μ_i $\rightarrow$ the minimum price which is the lower boundary for the i^{th} parking place.

$D_{f\,i}$ $\rightarrow$ the maximum possible demand for the i^{th} parking place.

C_i $\rightarrow$ the total running cost for the i^{th} parking place.

D_i $\rightarrow$ the total demand for the i^{th} parking place.

$\overline{P_{ij}}$ $\rightarrow$ the tolerance amount that people will go for parking-j who would prefer parking-i for saving the given tolerance amount.

3.1.2 The profit function for two parking.

Based on the current time, the management of parking-i may need to set the price for the next δ period of time to maximize their profit $\mathcal{P}_i$. As discussed in Section 2.2:

$$\mathcal{P}_i = I_i - C_i \tag{3.1.1}$$

where I_i is the total income for the next δ period of time and is given by:

$$I_i = P_i(N_i - q_i) \tag{3.1.2}$$

That simply is the product of the unit price by the number of occupied parking slots.

$$\text{where} \quad P_i = \ min\left\{(q_i - N_i)\left(\frac{P_m - \mu_i}{q_0 - N_i}\right) + \mu_i + \left(\frac{P_m - \mu_i}{D_{f\,i}}\right)D^i,\ P_m\right\} \tag{3.1.3}$$

Hence the profit function of the parking-i is given by:

$$\mathcal{P}_i = \left[min\left\{(q_i - N_i)\left(\frac{P_m - \mu_i}{q_0 - q_i}\right) + \left(\frac{P_m - \mu_i}{D_{f\,i}}\right)D^i + \mu_i,\ P_m\right\}\right](q_i - N_i) - C_i \tag{3.1.4}$$

where $i \ = \ 1, 2$

In order to express the profit function as a function of the price, we need to express demand in terms of the prices of the two parking places.

3.2 Demand as a function of prices

As demonstrated in Fig 3.1, the demand for a parking place can have three categories. These are people interested in parking-i, people interested in parking-j but will choose parking-i based on the price and people who have the same preference to both parking places.

Hence, the demand for parking-i can be given by $D^i = D^i_i + D^i_j + D^i_k$

where

- D^i_i with $i = 1, 2$, then the demand D^i_i are the people who are willing to use the parking i as long as it price does not exceed $P_j + \overline{P_{ii}}$.
- D^i_j with $i, j \in \{1, 2\}$ and $i \neq j$, then the demand D^i_j are the people who prefer parking-j but uses parking-i when the parking price in parking-i is less than $P_j - \overline{P_{ij}}$
- D^i_3 with $i = 1, 2$, then the demand D^i_3 are the people who are willing to use any parking as long as it is cheaper than the other.

Based on demand prediction and reading, suppose the the total demand in the region D is known with its components of D^i, where D^i_i, D^i_j and D^i_3 are demand where preference of parking is given to parking 1, 2 and some preference, respectively.

$D^i = D^i_i + D^i_j + D^i_3$

Each of the above three conditions are discussed below:

(A) Case one:

The total demand interested in parking - i is D^i_i. But this number will reduce if P_i goes beyond $P_j + \overline{P_{ii}}$, i.e. the if $P_i = P_j$ all this demand remains the same but when it keep on increasing it diminishes and become zero after $P_j + \overline{P_{ii}}$. Its graph is given below:

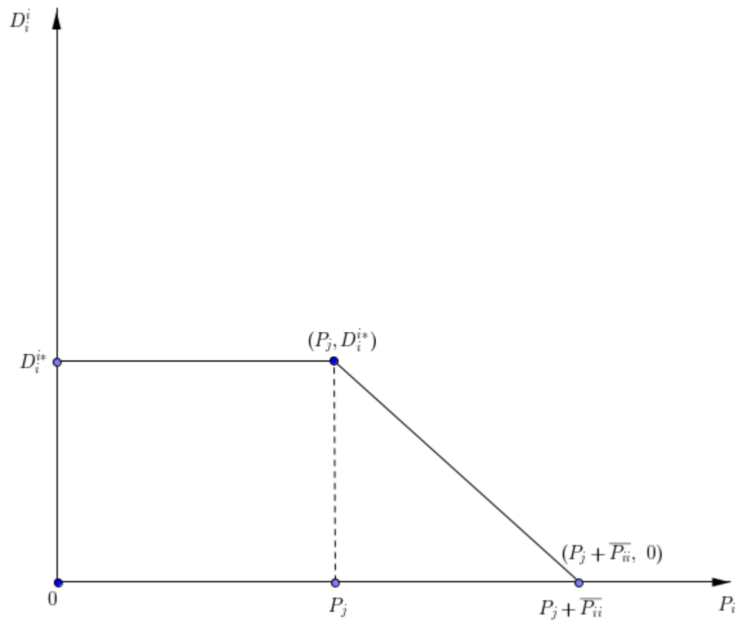

Figure 3.2: The 1st demand vs price plot for condition 1

Where D^{i*}_i is a maximum demand value of the demand function D^i_i. Hence, the demand curve can be expressed by:

$$D_i^i = \begin{cases} D_i^{i*} & \text{if} \quad P_i \leq P_j \\ f_{ii} & \text{if} \quad P_j \leq P_i \leq P_j + \overline{P_{ii}} \\ 0 & else \end{cases} \tag{3.2.1}$$

where f_{ii} is the line joining (P_j, D_i^{i*}) and $(P_j + \overline{P_{ii}}, 0)$.

Hence, the demand can be given by:

$$D_i^i = \begin{cases} D_i^{i*} & \text{if} \quad P_i \leq P_j \\ \frac{D_i^{i*}}{\overline{P_{ii}}}(\overline{P_{ii}} + P_j - P_i) & \text{if} \quad P_j \leq P_i \leq P_j + \overline{P_{ii}} \\ 0 & else \end{cases} \tag{3.2.2}$$

(B) Case two

The second case for parking-i, is a case in which it attracts customers who prefer parking-j (D_j^{i*}) by reducing the price by at least $\overline{P_{ij}}$. That means if P_i is less than by $\overline{P_{ij}}$ or more from P_j then the parking-j customers will choose parking-i. However if this price increases gradually to P_j the customers who will prefer parking-i will also reduced to zero. The scenario is demonstrated in the following figure:

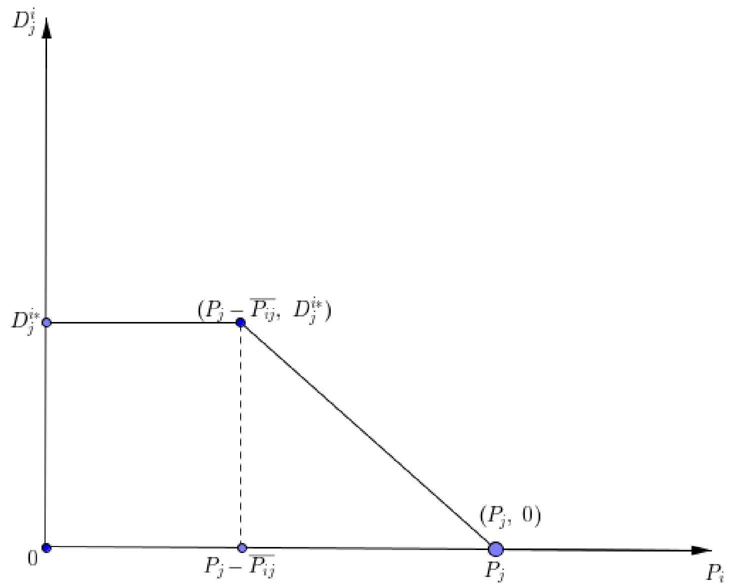

Figure 3.3: The 1st demand vs price plot for condition 2

Where D_j^{i*} is a maximum demand value of the demand function D_j^i. Equation for the curve can be given by:

$$D_j^i = \begin{cases} D_j^{i*} & \text{if} \quad P_i \leq P_j + \overline{P_{ij}} \\ f_{ij} & \text{if} \quad P_j - \overline{P_{ij}} \leq P_i \leq P_j \\ 0 & else \end{cases} \tag{3.2.3}$$

where f_{ij} refes to the line joining $(P_j - \overline{P_{ij}}, D_j^{i*})$ and $(P_j, 0)$. Hence, D_j^i is given by:

$$D_j^i = \begin{cases} D_j^{i*} & \text{if} \quad P_i \leq P_j + \overline{P_{ij}} \\ \frac{D_j^{i*}}{\overline{P_{ij}}}(P_j - P_i) & \text{if} \quad P_j - \overline{P_{ij}} \leq P_i \leq P_j \\ 0 & else \end{cases} \tag{3.2.4}$$

(C) Case three

The last case is regarding the demand which looks for the cheap parking without any preference to either of the parking places. The assumption here is that, if both parking places have the same price then they will share this demand equally. However, a unit increase in the price of parking-i will reduce its share of the demand to zero and reducing the price by a unit will increase its share of this demand to the maximum. The figure below demonstrates the scenario.

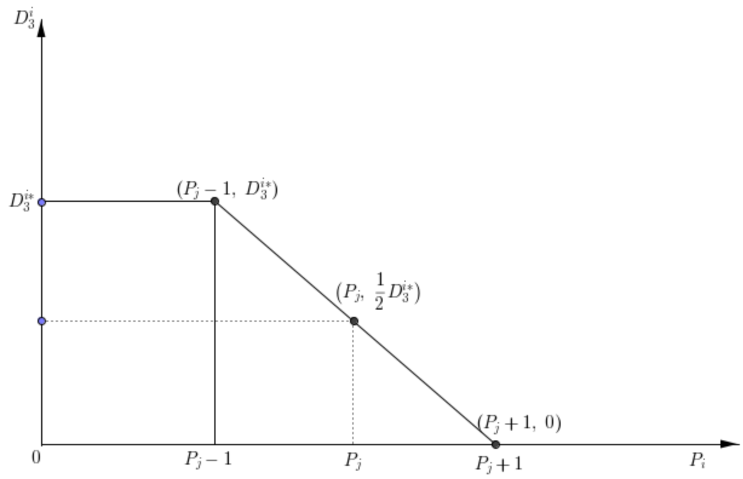

Figure 3.4: The 1st demand vs price plot for condition 3

Where D_3^{i*} is a maximum demand value of the demand function D_3^i, and then the given curve can be expressed by:

$$D_3^i = \begin{cases} D_i^{i*} & \text{if} \quad P_i \leq P_j - 1 \\ f_{i3} & \text{if} \quad P_j - 1 \leq P_i \leq P_j + 1 \\ 0 & else \end{cases} \tag{3.2.5}$$

where f_{i3} is a line joining $(P_j - 1, D_3^{i*})$ and $(P_j + 1, 0)$. Hence D_3^i is given by:

$$D_3^i = \begin{cases} D_3^{i*} & \text{if} \quad P_i \leq P_j - 1 \\ \frac{1}{2} D_3^{i*}(P_j - P_i + 1) & \text{if} \quad P_j - 1 \leq P_i \leq P_j + 1 \\ 0 & else \end{cases} \quad (3.2.6)$$

With this, it is possible to summarize and write the demand of a parking place depending on the price of the two parking places.

For example for parking 1, the demand will be given by

$D^1 = D_1^1 + D_2^1 + D_3^1$

where

$$D_1^1 = \begin{cases} D_1^{1*} & \text{if} \quad P_1 \leq P_2 \\ \frac{D_2^{1*}}{P_{11}^*}(P_{11}^* + P_2 - P_1) & \text{if} \quad P_2 \leq P_1 \leq P_2 + \overline{P_{11}} \\ 0 & else \end{cases} \quad (3.2.7)$$

$$D_2^1 = \begin{cases} D_2^{1*} & \text{if} \quad P_1 \leq P_2 + P_{12}^* \\ \frac{D_2^{1*}}{P_{12}^*}(P_2 - P_1) & \text{if} \quad P_2 - P_{12} \leq P_1 \leq P_2 \\ 0 & else \end{cases} \quad (3.2.8)$$

$$D_3^1 = \begin{cases} D_3^{1*} & \text{if} \quad P_1 \leq P_2 - 1 \\ \frac{1}{2} D_3^{1*}(P_2 - P_1 + 1) & \text{if} \quad P_2 - 1 \leq P_1 \leq P_2 + 1 \\ 0 & else \end{cases} \quad (3.2.9)$$

3.3 Bi-level formulation of pricing problem

Based on the formulation given, each parking management want to maximize the profit by fixing their price. However, the profit depends not only their price but also the price set by the other parking management. Hence, hierarchical decision making approach with two decision makers is a suitable model for the problem.

Without losing generality we can set the decision maker for parking place 1 to be the leader.

The optimization formulation then can be given by:

$$\begin{aligned} &\underset{P_1}{\text{Maximize}} && \mathcal{P}_1(P_1, P_2) \\ &\text{subject to} && P_2 \text{ solves },\underset{P_2}{\text{Maximize}}\ \mathcal{P}_2(P_1, P_2), \\ & && \mu_i \leq P_i \leq P_m. \end{aligned} \quad (3.3.1)$$

where P_1 and P_2 are given by Eq. 3.1.3.

The leader, i.e. the decision maker for parking-1, controls setting up the price for its parking but its profit will be affected not only by P_1 but also by P_2. But P_2 is controlled by the follower, i.e. the decision maker for parking-2. Hence, it is a continuous bi-level optimization problem.

Multi-level programming problems are one of the challenging optimization problems. Finding a rational reaction set, or parametric solution is not always an easy task (Oduguwa and Roy, 2002). Metaheuristic algorithms are becoming useful in dealing with challenging problems towards finding and approximate and acceptable solution. A solution approach inspired by natural adaptation based on (1+1)-evolutionary computing was proposed and used to solve multi-level problems Tilahun et al. (2012). In the solution approach, the leader will choose a value to its variable in order to maximize its gain, that solution then will be forwarded to the immediate follower. The follower will optimize its objective by fixing the upper level values. The last follower then forward the results to the leader and the iteration will go on until a termination criteria, mostly maximum number of iteration, is fulfilled. In each of these steps evolutionary strategy is used.

(1+1)-evolutionary strategy starts by randomly generating feasible solutions. The solutions will be updated by randomly moving it in its neighbourhood. The update is accepted if it is feasible and does better than the previous solution (i.e. its parent) otherwise the update will be rejected. The algorithm can be summarized as follows:

Table 3.1: (1+1)-Evolutionary Strategy

1	Input:	Parametric	Values
2		λ	Step length
3		MaxGen	Maximum number of iteration
4		M	Population size
5		$f(x)$	Objective function
6		S	Feasible region
7	Algorithm:		
8		x_i:	randomly generate M feasible solutions
9		for i=1:M	
10			$y_i = x_i + \lambda * (rand - 0.5)$
11			If y_i is feasible and $f(y_i) \geq f(x_i)$ then $x_i := y_i$
12		end for	
13		If termination	criteria is fulfil terminate else go back to step 9

4. Numerical Results

In this section, a hypothetical situation is discussed to simulate and demonstrate how the parking pricing and competition between two parking management occurs.

4.1 Set-up

4.1.1 Software.

The software used is MATLAB Online R2019b (9.7.0) Version 19.0 at https://matlab.mathworks.com, and all the MATLAB codes used are available at https://github.com/nkos09amahle/optimizing-parking-profit by Tilahun (2019) .

4.1.2 Algorithm parameter setting.

The codes used includes two objective functions (profit functions), for each of the parking owner as a function of the price set by each of the owners. However, each parking owner can tune or control the price set by him/herself only.

The main function called *simu* in the code or Matlab uses the $(1+1)$ evolutionary strategy to generate a better solution using an evolutionary strategy based algorithm inspired by natural adaptation. In both profit function there is a demand function within them, and also in each demand function there are three function of demand under it, since each demand per profit is made out of three conditions.

The $(1+1)$ evolution strategy on main function *simu* has the input parameters which are step length $\lambda = 25.5$, maximum iteration number for the evolutionary strategy, $Gen = 100$ and iteration number for the adaptation based algorithm $MaxGen = 25$. With the above parameters each and every iteration, a solution will be produced for both profit-1 and profit-2.

4.2 Data

The hypothetical example used has two parking places with capacity of 60 and 70 which is for parking-1 and parking-2 respectively, where both places have the upper maximum and lower minimum boundary price which is $P_m = R50$ and $mu = R5$. There are 40 and 25 people who prefer to use the first parking and the second parking respectively, while there are also 5 people who are neutral and who prefers to choose the lowest priced parking between the two.

The available parking lot,for each parking-1 and parking-2 respectively is 45 and 50 which will change after period of time, but the minimum amount of the available q_0 is equals one. Maximum parking demand in the area is assumed to be 100. The data is summarized in Table 4.1:

	μ	P_m	q_0	q_i	N_i	D_{f_i}	D_i
Parking $i = 1$	5	50	1	45	60	100	40
Parking $i = 2$	5	50	1	50	70	100	25

Table 4.1: Small Data

The simulation is also conducted by changing the demand in both parking to 25 for parking-1 and 37 for parking-2.

4.3 Results

The simulation was conducted according to the setup discuss in the previous section. Since metaheuristic based algorithm is used, multiple runs were conducted to provide similar results. Hence, below some of the results are listed.

4.3.1 Firsts Result.

By using the unchanged data from the table 4.1 into our code, we will get these following results in figure 4.1 below:

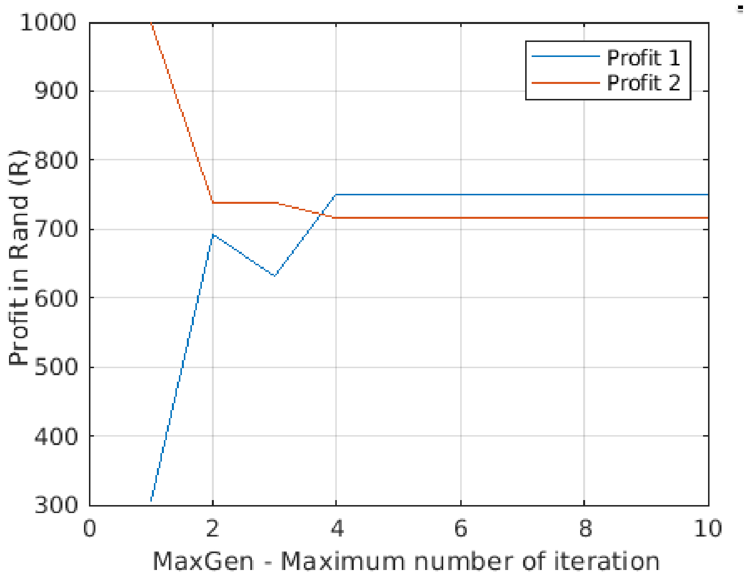

Figure 4.1

From the plot above,profit maximization is illustrated for the owner of parking lot 1.We see that as he profit line 1 increases,profit line 2 tends to decrease,illustrating that the demand for parking lot 1 is higher in demand than that of parking lot 2,resulting in profit maximization of parking lot 1. The more customers occupy parking lot 1 it results to fewer customers occupying parking lot 2,resulting in the plot reaching to a point of maximization and equilibrium.

4.3.2 Second Result.

In order to obtain the following result we used the same data from table 4.1 but with a different demand for the second parking as mentioned below table 4.1. There were many results obtained which were of the same appearance but the most suitable result is illustrated in figure 4.2 below.

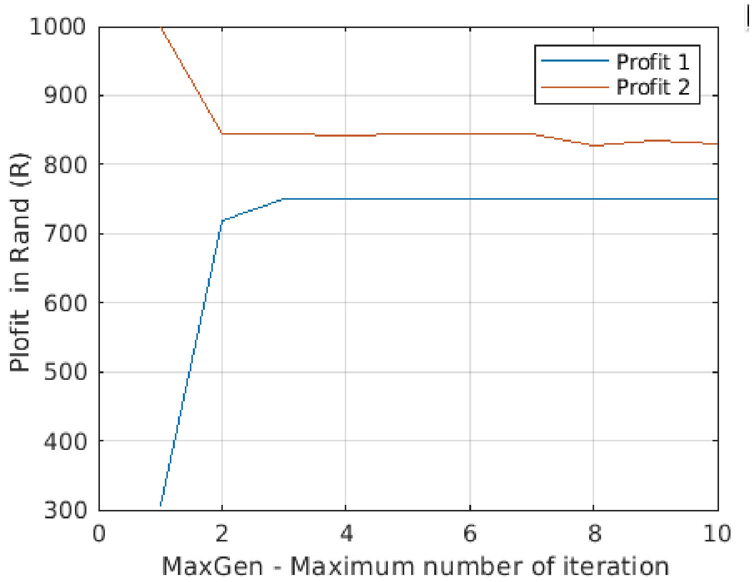

Figure 4.2

In the above plot in figure 4.2 there is almost the same result that were obtain in the first problem in figure 4.1. The plot can be seen that when the line of profit 1 is increases, the line for profit 2 decreases. And we also know that the demand of the first parking lots is almost equals to the demand of the second parking lots, but it more with an addition of 3. The more customers or demand in parking one, is the fewer for two, so it make sense if the graph is showing lines approaching each other up until they reach almost the same maximum profit. According to the plots in figure 4.2 it may be noted that parking-1 can not exceed profit maximization of parking-2.

5. Discussion

The aim of this thesis was to develop a pricing parking model depending on the predicted demand towards the available space which can lead to gaining the maximum profit. Here bi-level optimization model is used to calculate and show the interaction between the two competitor's decisions with regards to obtaining the optimum profit. The first objective was to find the relationship between the demand for off-street relative to the rate of prices implemented, by creating a model for pricing using the inverse economic relation of quantity and price. This is later used in the construction of the profit function based on demand prediction and available space with a constant cost.

The study also considered the situation where there are two parking place owners who are business competitors and each of these owners want to maximize their profit, but the decision of one owner will affect the profit of other by affecting the demand. The demand function in terms of price were derive using the assumed conditions, where each general demand function is divided into three demand functions which are the conditions of preference of drivers.

Natural adaptation inspired algorithm based on evolutionary strategy is used used to solve the formulated bi-level problem. Simulation based illustration was also presented.

This research considered two parking places, generalizing the discussion to multiple parking places is one possible future work. A linear curve is used to model price demand relation, studying the effect of other curves can also be done in the future. A more detailed simulation analysis based on complicated and large data should also be done in the future along with testing the approach on a real scenario.

Acknowledgements

I would like to thank and acknowledge the AIMS program, AIMS director Prof. Barry Green, academic director Dr. Simukai Utete, my tutor Prudence Djagba and all AIMS staff members who were helpful and willing to assist by all necessary means for me to get my work done, my supervisor Dr. Surafel Tilahun who has been of great assistance towards my research, discovering new things related to my project and I am delighted to have worked with him, my church family which includes the Mkhwanazi chief (iNkosi M. Mkhwanazi) with his officers for the support and encouragement they have given me, including church officials whom I would not have made it thus without their support and prayers, I am also thankful to my church brothers and sisters who were helpful and supportive towards my journey, my physical family whom are grandma Mrs. Dorah Dube, Zamokuhle Mthethwa and Zanele Dube for their love and care, as well as my very special loving woman who has been supportive to me, my girlfriend miss Amy L. Pillay. Thank you, everyone, who made this possible.

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