Optimisation of the Power Generation Functional for Wind Turbine Hub Height

Bijoue Meslye Schailde Obanda (bijoue@aims.ac.za)
African Institute for Mathematical Sciences (AIMS)

Supervised by: Prof Nawaz Mahomed
Stellenbosch University, South Africa

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Abstract

This essay is aimed at investigating the optimal hub height for maximum power generation. The impact of hub height on the power generation is studied using wind speed data and two wind turbines of $80kW$ and $11kW$ rated power. We use the conservation of momentum integral, law of mass conservation and Bernoulli's theorem to establish the mathematical model of the power generated. The analytical equation obtained is solved using a numerical method. The results show that the maximum power is generated at the highest height.

Keywords: Optimisation, Power Generation, Wind Turbine, Hub Height

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Bijoue Meslye Schailde Obanda, 14 May 2020

Contents

1. Introduction

The production of electricity from renewable resources is an alternative to fossil fuels and an unique current solution to fight against global warming. Therefore, studies on renewable energies have become interesting fields due to their clean and renewable nature. There are many types of renewable energy: wind, solar, geothermal, marine, hydropower and biomass energies (Panwar et al., 2011).

This project is focused on wind energy. Wind produces electricity using a wind turbine. The energy produced by a wind turbine is variable and is influenced by many parameters. These parameters include the choice of the wind turbine which will use the best energy resource available at the given site and the hub height positioning, which is the distance of the rotor of a wind turbine from the ground. The hub height infers the wind speed required for a wind turbine to operate. Thus the optimisation of hub height is necessary to maximise the power generated. This essay will focus on the wind speed data obtained at Kafue Gorge in Zambia and will use two wind turbines, Enercon E-18 and Gaia-wind to optimise the hub height ((Lucas and Silvio), (Huskey et al., 2009)).

1.1 Motivation

Wind energy is a clean and renewable resource usually used in rural areas to produce electricity. Therefore, when used on a large scale, wind energy contributes to a decrease in greenhouse gases in the atmosphere. However, the power generated from this source of energy is very variable due to seasonal variation of wind speed; the choice of the wind turbine size that best exploits the wind resource at the a given site; as well as the hub height position. Thus the optimisation of hub height is required to improve wind turbine power output. Many studies are undertaken in this field. However, most of them have an objective oriented to minimise the cost. Hence, this essay is aimed at optimising the hub height for a maximum power output of a wind turbine.

1.2 Outline of the Research Project

The research project is organised as following: Chapter 2, presents related work including the methods used. In chapter 3, the mathematical model of the power that must be generated by a wind turbine using wind speed power is established. Chapter 4, focuses on the experimental work and methods used. Chapter 5, contains the results obtained and the discussion. Finally, a general conclusion is presented in chapter 6.

2. Backgound

In this section, we briefly present work related to the optimisation of power generation for wind turbine due to hub height including the different methods used and the results obtained.

As previously stated, many studies have been presented on hub height optimisation, many of which focused on minimising costs. Ahmadreza and Cristina (2017) studied the influence of optimising hub height on the energy produced in one year for a wind farm. They considered only one parameter variable, the hub height of each wind turbine. Firstly, two wind turbines were disposed in the direction of the wind, one with a stationary hub height and the other with a variable hub height. It was noticed that the wind turbine with the variable hub height produced more energy in comparison to the wind turbine with the stationary hub height. Secondly, they compared the Annual Energy Production (AEP) of the wind turbines in two sites. The hub height of the wind turbines in each site were $80m$ and, $100m$ and $57{,}4m$ respectively. It was observed that the AEP in the site with a different hub height was 2% greater. The PARK model (a tool used to evaluate the Annual Energy Production) was used to optimise the results.

Jaehwan et al. (2014) developed a methed to optimise the hub height of a wind turbine. They defined a function called an "Objective function" which represents annual net profit given by

$$Obj = C_e AEP - C_{AOM} AEP - \frac{ICC}{n}. \tag{2.0.1}$$

where different terms are defined as follows

ICC is the Initial Capital Cost,

n is the lifetime of the wind turbine,

C_{AOM} is the operating and maintenance costs per kWh,

C_e is the cost of electricity expressed in the function of hub height,

AEP represents the Annual Energy Produced by the wind turbine.

The Annual Energy Produced is calculated using Equation (2.0.2)

$$AEP = T \int_{v_{\text{cut-in}}}^{v_{\text{cut-out}}} \frac{1}{2} \rho C_p v^3 \pi R^2 p(v) dv. \tag{2.0.2}$$

In this Equation, T is the time in one year, ρ is air density, v is the wind speed, C_p is the power coefficient of the wind turbine used, R is the rotor diameter and $p(v)$ is the wind speed distribution.

The wind speed distribution $p(v)$ is defined by the Rayleigh distribution as shown in the following equation

$$p(v) = \frac{\pi v}{2\overline{v}^2} e^{\frac{\pi}{4}\left(\frac{v}{\overline{v}}\right)^2}. \tag{2.0.3}$$

The logarithmic law given in Equation (2.0.4) is used to express the wind speed in terms of hub height. In this Equation $\overline{v}$ represents average wind speed, z_0 is the roughness and h_{ref} is the reference height.

$$\overline{v}(h) = \overline{v}(h_{ref}) \frac{ln\left(\frac{h}{z_0}\right)}{ln\left(\frac{h_{ref}}{z_0}\right)}. \tag{2.0.4}$$

To optimise the hub height, Jaehwan et al. (2014) differentiated the objective function with respect to h and equated it to zero. They find an interval of h where AEP is zero, thus the optimal hub height is determined by the value of AEP maximum in this interval.

In addition, Jung-Tae et al. (2019) proposed a method to determine the optimum hub height. In the process they found the Annual Energy Production (AEP) from the wind energy resource of South Korea for the hub heights from $40m$ to $100m$. They also presented a graph of the cost of energy (COE), thus optimal hub height corresponded to low costs of energy production. The AEP losses from the decreasing of hub height was replaced by increasing rotor diameter.

Ying et al. (2015) developed an algorithm to optimise the wind park position aimed at increasing the energy production. The algorithm was applied on a wind park multi-hub height wind turbine. Thus the power output was improved due to hub height optimisation.

3. Mathematical Model

3.1 Wind Energy Calculation and Maximum Power Extracted by Wind Turbine

In this Chapter, we present the mathematical model of wind power and the theoretical maximum power that can be extracted by a wind turbine. The flow of wind through the rotor looks like as shown in Figure 3.1 and is considered as the flow of a viscous incompressible fluid. Thus the fluid mechanics equations applied in incompressible fluids will apply.

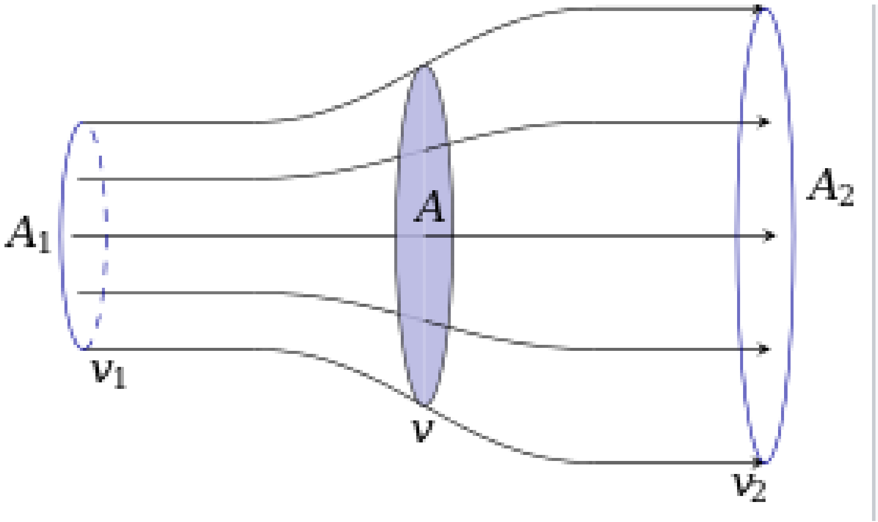

Figure 3.1: Illustration of the wind profile through a wind turbine (Wikipedia contributors, 2019)

In classical mechanics the energy of a moving particle is given in the following equation

$$E = \frac{1}{2} m V_1^2. \tag{3.1.1}$$

The flow of wind is a flux of a higher number of particles through an area, where the mass is defined by

$$m = \rho V_1 A t. \tag{3.1.2}$$

Kinetic energy of wind is obtained by combining Equation (3.1.1) and (3.1.2)

$$E = \frac{1}{2} \rho A V_1^3 t. \tag{3.1.3}$$

Hence, the kinetic power content of the undisturbed upstream wind velocity is

$$P = \frac{E}{t} = \frac{1}{2} \rho A V_1^3 \qquad ((\text{Wass}, 2018), (\text{Perkin}, 2014)). \tag{3.1.4}$$

A wind turbine generates electricity through the kinetic energy of wind. However, there is a theoretical maximum power that can be extracted from the wind, independent of the design of the wind turbine in open flow. As the air is an incompressible fluid, the conservation mass equation can be written as

$$V_1 A_1 = V_2 A_2 = VA. \tag{3.1.5}$$

Conservation of horizontal momentum integral

$$\int_{\partial v} \left[\rho\left(\vec{V}.\vec{n}\right)\vec{V} + \left(\vec{P}\vec{n}\right)\right] ds = 0. \tag{3.1.6}$$

The force on the area A is the pressure force defined as

$$F = \int_{\partial v} \vec{P}\vec{n} ds.$$

Using Equation (3.1.6), the pressure force becomes

$$F = -\int_{\partial v} \rho\left(\vec{V}.\vec{n}\right)\vec{V},$$

$$F = \rho\left[V_1^2 A_1 - V_2^2 A_2\right].$$

By substituting (3.1.5), the pressure force becomes

$$F = \rho V A\left(V_1 - V_2\right).$$

Hence, the extractable power by the wind turbine is obtained as

$$P = F \times \frac{d}{T} = F \times V,$$

$$P = \rho V^2 A\left(V_1 - V_2\right). \tag{3.1.7}$$

Since on the area A there is turbulence, Bernoulli's theorem can be applied along the stream line before and beyond the rotor (John and William, 2013).

$$H_1 = \frac{1}{2}\rho V_1^2 + P_1,$$
$$H_2 = \frac{1}{2}\rho V_2^2 + P_2,$$

Involving the change of energy density through the rotor gives

$$H_1 - H_2 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right).$$

Extractable power in terms of the energy density becomes

$$P = \left(H_1 - H_2\right) V A,$$

$$P = \frac{1}{2}\rho V A \left(V_1^2 - V_2^2\right). \tag{3.1.8}$$

The velocity at the rotor can be obtain from (3.1.7) and (3.1.8) as

$$V = \frac{1}{2}\left(V_1 + V_2\right). \tag{3.1.9}$$

Then substituting into the wind turbine power, the velocity at the wind rotor V gives

$$P = \frac{1}{4}\rho A \left(V_1^2 - V_2^2\right)\left(V_1 + V_2\right).$$

The extractable power expressed in terms of the interference factor or downstream velocity factor

$$b = \frac{V_2}{V_1},$$

is

$$P = \frac{1}{4}\rho V_1^3 A \left(1 - b^2\right)\left(1 + b\right). \tag{3.1.10}$$

The power coefficient of the wind turbine C_p is defined as the ratio of the wind turbine and wind power

$$C_p = \frac{P}{P_w},$$

$$C_p = \frac{1}{2}\left(1 - b^2\right)\left(1 + b\right).$$

The maximum value of power coefficient is given by differentiating the power coefficient with respect to b and equating it to zero (John and William, 2013).

$$\frac{dC_p}{db} = \left(1 - 3b\right)\left(1 + b\right) = 0. \tag{3.1.11}$$

Equation (3.1.11) has two solutions: $b = -1$ and $b = \frac{1}{3}$.

The first solution $b = -1 \Rightarrow V_1 = -V_2$, is trivial.

The second solution

$$b = \frac{V_2}{V_1} = \frac{1}{3} \Rightarrow V_2 = \frac{1}{3}V_1 \tag{3.1.12}$$

means that for an optimal operation, the downstream velocity V_2 is equal to one third of the upstream velocity V_1.

Then the maximum value of power coefficient C_p is obtained when $b = \frac{1}{3}$, thus giving us

$$C_p = \frac{1}{2}\left(1 - \left(\frac{1}{3}\right)^2\right)\left(1 + \frac{1}{3}\right),$$

$$C_p = \frac{16}{27} = 0.59259. \tag{3.1.13}$$

Equation (3.1.13) represents the theoretical maximum percent extractable power from the wind, known as Betz Limit (John and William, 2013). Generally, the wind turbines produce below this limit.

The substitution of Equation (3.1.12) into (3.1.10) gives the theoretical maximum extractable power

$$P_{max} = \frac{1}{2}\rho A C_p V_1^3 = C_p P_w.$$

3.2 Power Generated by Wind Turbines

The theoretical maximum power generated by wind turbines is obtained by multiplying the extraction power with the turbine efficiency η. It's value lies between the cut-in speed V_a and the cut-out speed V_b. Hence, the total power is the summation in this interval. Where η and C_p are both functions of wind velocity.

$$P = \eta \times P_{max} = \frac{1}{2}\rho A \eta C_p V_1^3,$$

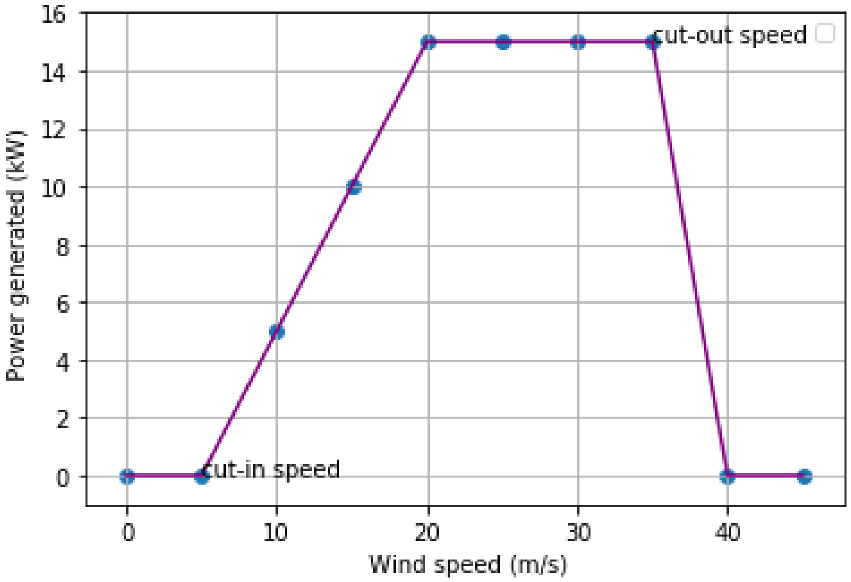

Figure 3.2: Power generated by wind turbine

$$P_T = \int_{V_a}^{V_b} \frac{1}{2}\rho A \eta(V_1) C_p(V_1) V_1^3 p(V_1) dV_1. \tag{3.2.1}$$

where $p(V_1)$ is the probability density function of the wind. The probability density function is used to describe wind speed distribution in a given site and to accurately assess the wind potential. It is obtained from Weibull distribution defined by

$$p(V_1) = \frac{k}{c}\left(\frac{V_1}{c}\right)^{k-1} \exp\left(-\left(\frac{V_1}{c}\right)^{k}\right). \tag{3.2.2}$$

where k is the shape parameter,

V_1 is the wind speed,

$$c = \frac{\overline{V_1}}{\Gamma\left(1+\frac{1}{k}\right)} \quad \text{is the scale factor,} \tag{3.2.3}$$

$\overline{V_1}$ is the annual average of wind speed,

Γ represents the Gamma function.

The parameters c and k can be determined using Equation (3.2.3) and also using the square of the standard deviation for the wind speed given by

$$\sigma^2 = \overline{V_1}^2\left[\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1\right] \qquad ((\text{John and William}, 2013), \quad (\text{Wass}, 2018)). \tag{3.2.4}$$

It is known that the wind speed increases with the height $(V_1(z))$, hence, Hellmann power law given below is used to introduce the hub height dependence in the theoretical maximum power generated (Jaehwan et al., 2015).

$$\frac{\overline{V_1}}{\overline{V_0}} = \left(\frac{z_1}{z_0}\right)^{\alpha}, \tag{3.2.5}$$

$$\Longrightarrow \overline{V_1} = \left(\frac{z_1}{z_0}\right)^{\alpha}\overline{V_0}. \tag{3.2.6}$$

There parameters are defined as follows

V_1 is the wind speed to height z_1,

V_0 is the wind speed to height z_0,

α is the wind shear coefficient or Hellmann exponent.

By substituting (3.2.2) into (3.2.1), the power generated becomes

$$P_T = \frac{1}{2}\rho A\int_{V_a}^{V_b} \eta(V_1)C_p(V_1)V_1^3\frac{k}{c}\left(\frac{V_1}{c}\right)^{k-1} \exp\left(-\left(\frac{V_1}{c}\right)^{k}\right)dV_1.$$

Using an approximation on efficiency

$$\eta(V_1) = \eta, \tag{3.2.7}$$

We have

$$P_T = \frac{1}{2}\rho A\eta\int_{V_a}^{V_b} C_p(V_1)V_1^3\frac{k}{c}\left(\frac{V_1}{c}\right)^{k-1} \exp\left(-\left(\frac{V_1}{c}\right)^{k}\right)dV_1. \tag{3.2.8}$$

3.3 Resolution of the Equation of the Power Generated

The equation of the power generated is resolved using rectangles method. The rectangles method is a numerical method which allows one to calculate the integral of a function $g(x)$ over an interval $[a, b]$. This method consists of divising the area under the curve $g(x)$ with *n* rectangles on the interval $[a, b]$. The solution obtained is an approximation solution and it tends to the real solution when *n* is large.

Hence

$$\int_a^b g(x)dx = \int_{a_0}^{a_1} g(x)dx + \int_{a_1}^{a_2} g(x)dx + ... + \int_{a_{n-1}}^{a_n} g(x)dx,$$

Then

$$\int_a^b g(x)dx = \sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} g(x)dx.$$

And the integral

$$\int_{a_i}^{a_{i+1}} g(x)dx = B \times H(\text{ Area of Rectangle}), \tag{3.3.1}$$

where B and H are the base and height respectively of rectangles.

$$H = g(a_i),$$
$$B = \frac{b-a}{n},$$

The i^{th} point x_i is defined as

$$x_i = a + i\frac{b-a}{n}. \tag{3.3.2}$$

Then the integral of $g(x)$ becomes

$$\int_a^b g(x)dx = \sum_{i=0}^{n-1} g(a_i)\frac{b-a}{n}.$$

The integral of the function $P_T(V_1)$ over interval $[V_{\text{cut-in}}, V_{\text{cut-out}}]$ defined in Equation (3.2.8) is solved using this algorithm. Figure 3.2 of the power output is divided into *n* points from $V_{\text{cut-in}}(V_a)$ to $V_{\text{cut-out}}(V_b)$.

The i^{th} wind speed V_{1i} is given by

$$V_{1i} = V_a + i\frac{V_b - V_a}{n}, \tag{3.3.3}$$

where

$$V_0 = V_a,$$
$$V_n = V_b.$$

The integral of $P_T(V_1)$ over $[V_a,V_b]$ becomes

$$\int_{V_a}^{V_b} P_T(V_1)dV_1 = \int_{V_0}^{V_1} P_T(V_1)dV_1 + \int_{V_1}^{V_2} P_T(V_1)dV_1 + ... + \int_{V_{n-1}}^{V_n} P_T(V_1)dV_1,$$

Then

$$\int_{V_a}^{V_b} P_T(V_1)dV_1 = \sum_{i=0}^{n-1} \int_{V_i}^{V_{i+1}} P_T(V_1)dV_1,$$

Using Equation (3.3.1), the function $P_T(V_1)$ becomes

$$\int_{V_a}^{V_b} P_T(V_1)dV_1 = \sum_{i=0}^{n-1} P_T(V_{1i})\frac{V_b - V_a}{n}, \tag{3.3.4}$$

By substituting Equation (3.3.4) in (3.2.8), the total power that must be generated by a given wind turbine becomes

$$P_T = \frac{1}{2}\rho A\eta \sum_{i=0}^{n-1} C_p(V_{1i})V_{1i}^3 \frac{k}{c}\left(\frac{V_{1i}}{c}\right)^{k-1} \exp\left(-\left(\frac{V_1}{c}\right)^k\right)\frac{V_b - V_a}{n}. \tag{3.3.5}$$

4. Experimental Work

This chapter is focused on experimental work to determine an optimal hub height. The research project uses the wind speeds data obtained at Kafue Gorge in Zambia. This data was furnished by a Thies anemometer in the South South-East direction. The wind speed data provided is from 07 February 2017 to 23 April 2019, taken every 10 minutes at heights of 40m, 50m and 60m, in order to measure wind speed dependence on the height. To exploit the data, Symphonie Desktop, Excel and Python were used.

4.1 Description of Wind Speed Data Site

Kafue Gorge is an area near to Lusaka, capital of Zambia. The geographic location is 15°48’44,19“S and 28°24’55,42”E. Wind is the movement of air (i.e particles, atoms and molecules) of the atmosphere between high pressure regions and low pressure regions. The wind speed is a parameter dependent on the atmospheric temperature. Hence its data changes over time, as can be seen in Figure 4.1 showing the wind speed measurement distribution at the heights 40m, 50m and 60m.

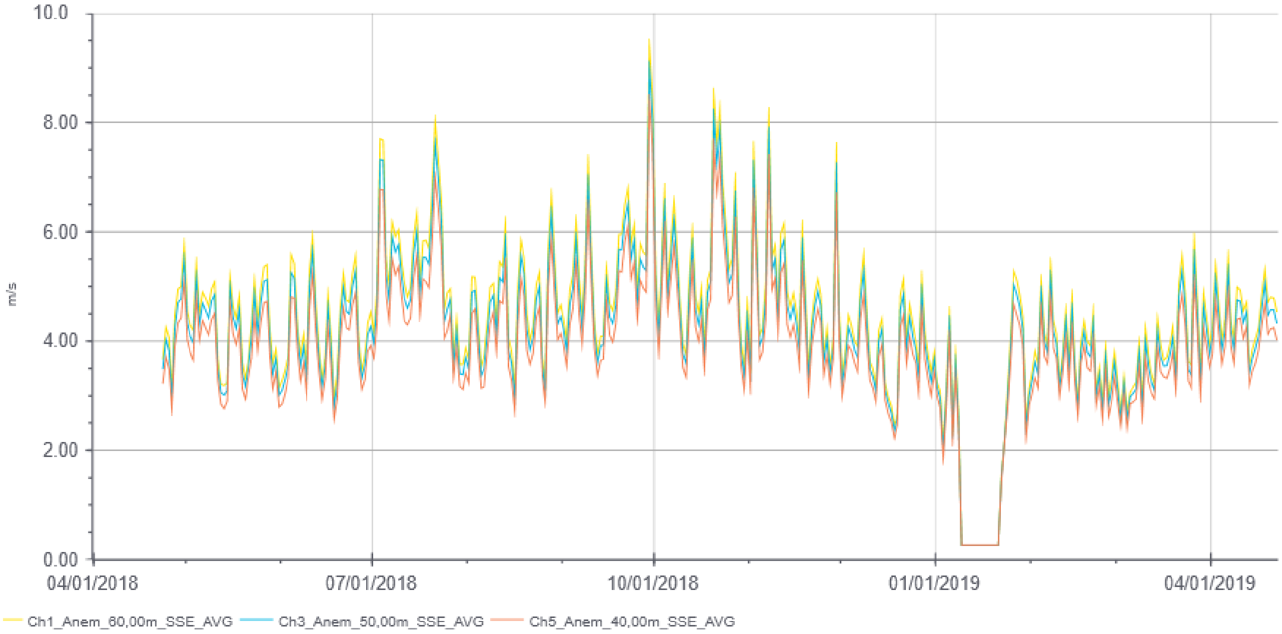

Figure 4.1: Wind speed measurement distribution at 40m, 50m and 60m

4.2 Description of Wind Turbine Used

To optimise the hub height which will maximise the power output of a wind turbine using wind speed data, two turbines are used in this project. The first wind turbine is Gaia-wind with a rated power of 11kW. It has two blades, the rotor diameter is 13m. The swept area of this turbine is 132,7m^2. Its cut-in and cut-out wind speeds are 3m/s and 19,92m/s respectively.

The second wind turbine used is an Enercon E-18, its rated power is 80kW, the rotor diameter is 18m, it has three blades. The swept area is 254,5m^2, its cut-in and cut-out wind speeds are 3,5m/s and

$25m/s$ respectively.

The maximum power coefficient of the Gaia-wind and the Enercon E-18 wind turbine are 29% and 44% respectively. Figures 4.2 to 4.5 present the power generated and power coefficient of these turbines.

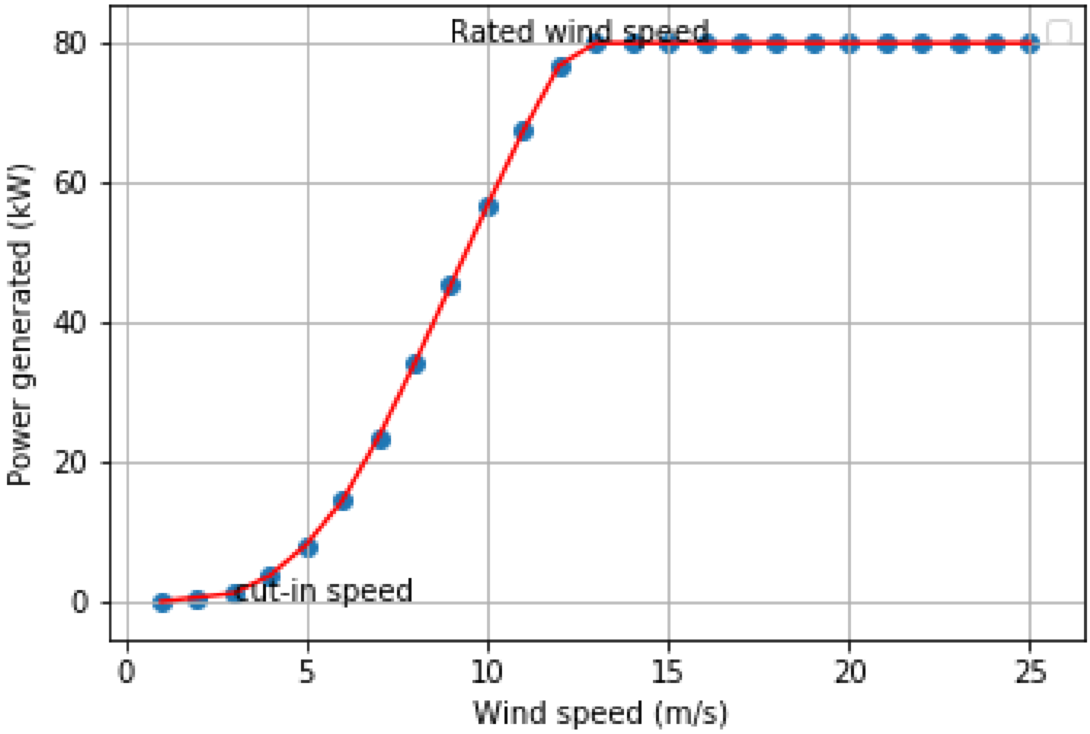

Figure 4.2: Power generated curve of Enercon E-18 wind turbine

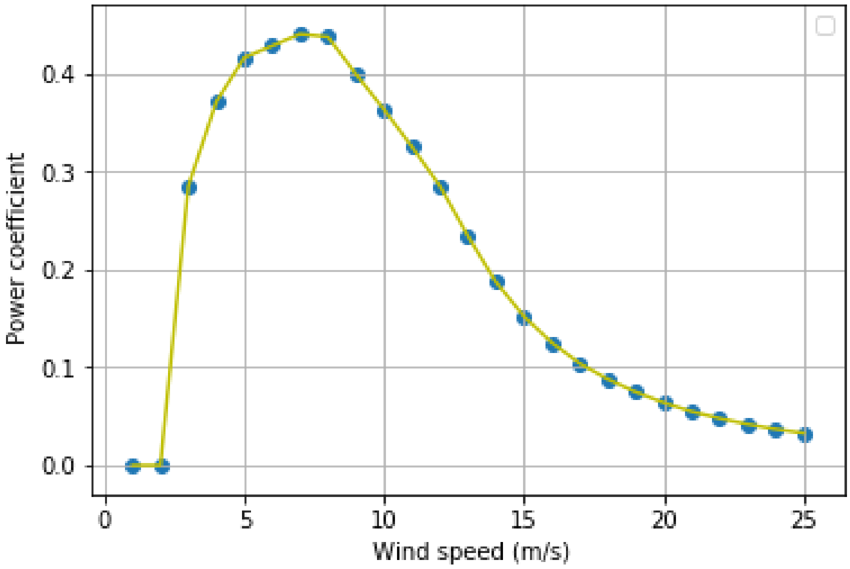

Figure 4.3: Power coefficient curve of Enercon E-18 wind turbine

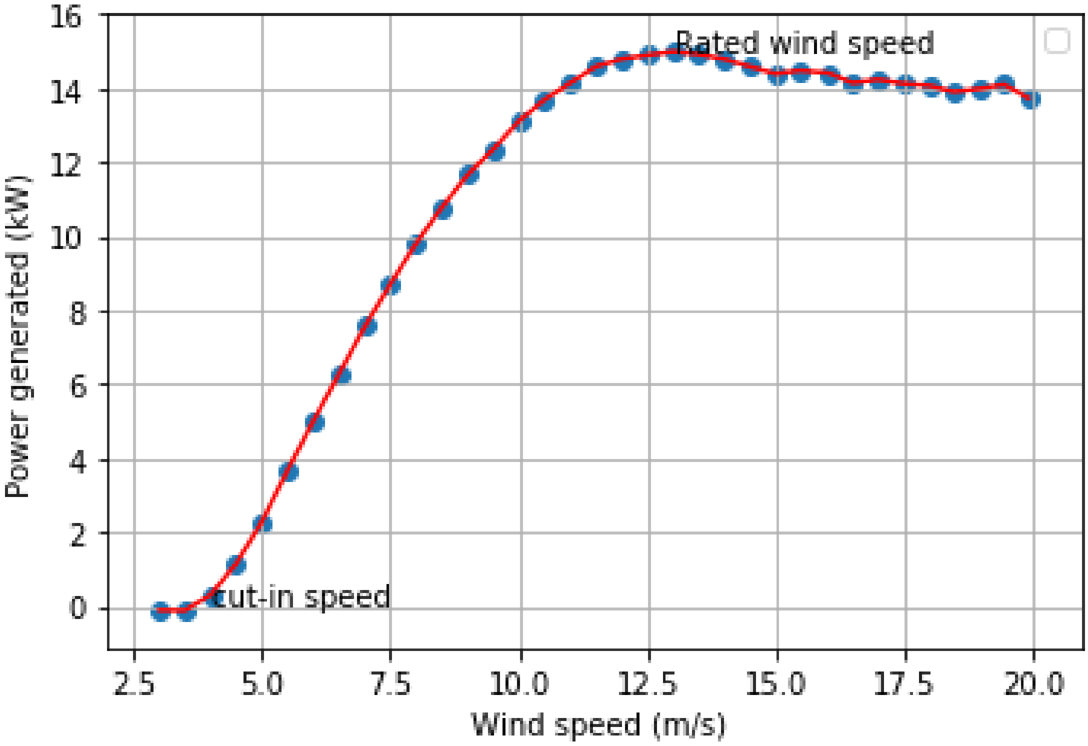

Figure 4.4: Power generated curve of Gaia-wind turbine

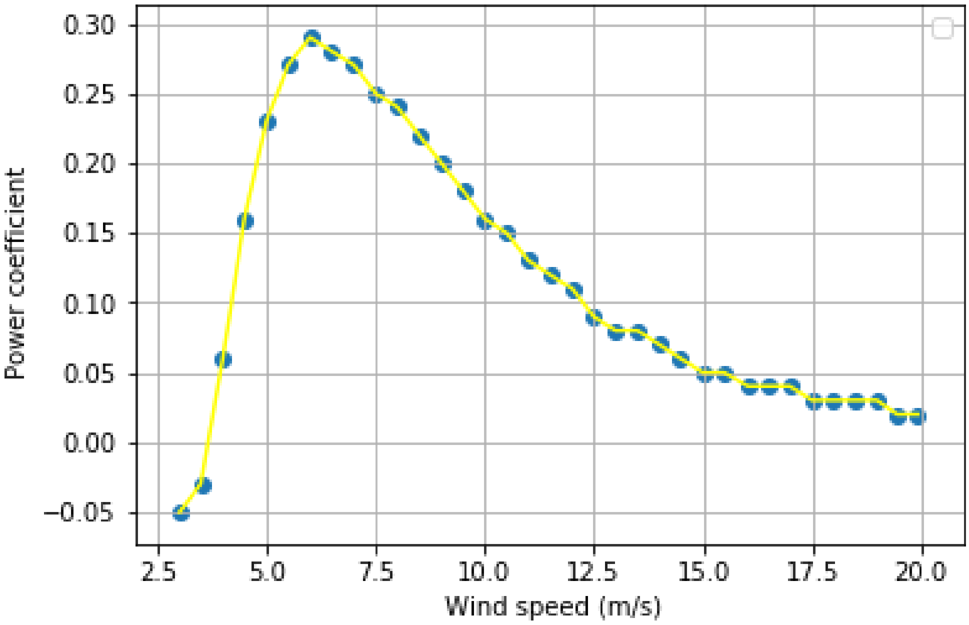

Figure 4.5: Power coefficient of Gaia-wind turbine

4.3 Methods

To evaluate the hub height which will give the maximum power output, the procedure is as follows.

Firstly, the Weibull parameters are determined using average wind speed and standard deviation computed automatilly from Symphonie Desktop at heights 40m, 50m and 60m. The calculations are computed by Equations (3.2.3) and (3.2.4), and the empirical formula of Lysen and Newton-Raphson method ((Camilo et al., 2014),(Prem et al., 2018)), to validate the results which are given in the Table 4.1.

Height (m)	40	50	60
Average wind speed $\overline{V}$ (m/s)	3.93	4,23	4,43
Standard deviation σ	0,73	0,71	0,70
Shape parameter k	6,222	6,946	7,417
Scale factor $c\,(m/s)$	3,656	3,958	4.159

Table 4.1: The average wind speed and standard deviation obtained from Symphonie Desktop

The Newton-Raphson method and Lysen formula are given in Equations (4.3.1) and (4.3.2) respectively

$$k = \left(\frac{\sigma}{\overline{V}}\right)^{-1.086}. \tag{4.3.1}$$

$$c = \overline{V}\left(0.568 + \frac{0.433}{k}\right)^{-\frac{1}{k}}. \tag{4.3.2}$$

Secondly, the wind speed data is exported into Excel. We determine the average wind speeds, standard deviations and Weibull parameters at heights 40m, 50m and 60m. Using power law, the wind shear coefficient (α) is determined at each height and the average wind shear coefficient is also calculated. The results of these experiments are presented in Table 4.2. Figures 4.6 to 4.8, present wind speed distribution experiment (Probability Density Distribution) and theory (Weibull function). These functions are combined.

Height (m)	40	50	60
Average wind speed $\overline{V}$ (m/s)	3,9039	4,2007	4,4067
Standard deviation (σ)	1,554	1,653	1,726
Shape parameter k	2,718	2,753	2,767
Scale factor $c\,(m/s)$	3,427	3,738	3,922
Wind shear coefficient α	0,3284	0,2988	0,2626
Average Wind shear coefficient $\overline{\alpha}$	0,2965		

Table 4.2: The average wind speed and wind shear coefficient at different heights calculated from Excel

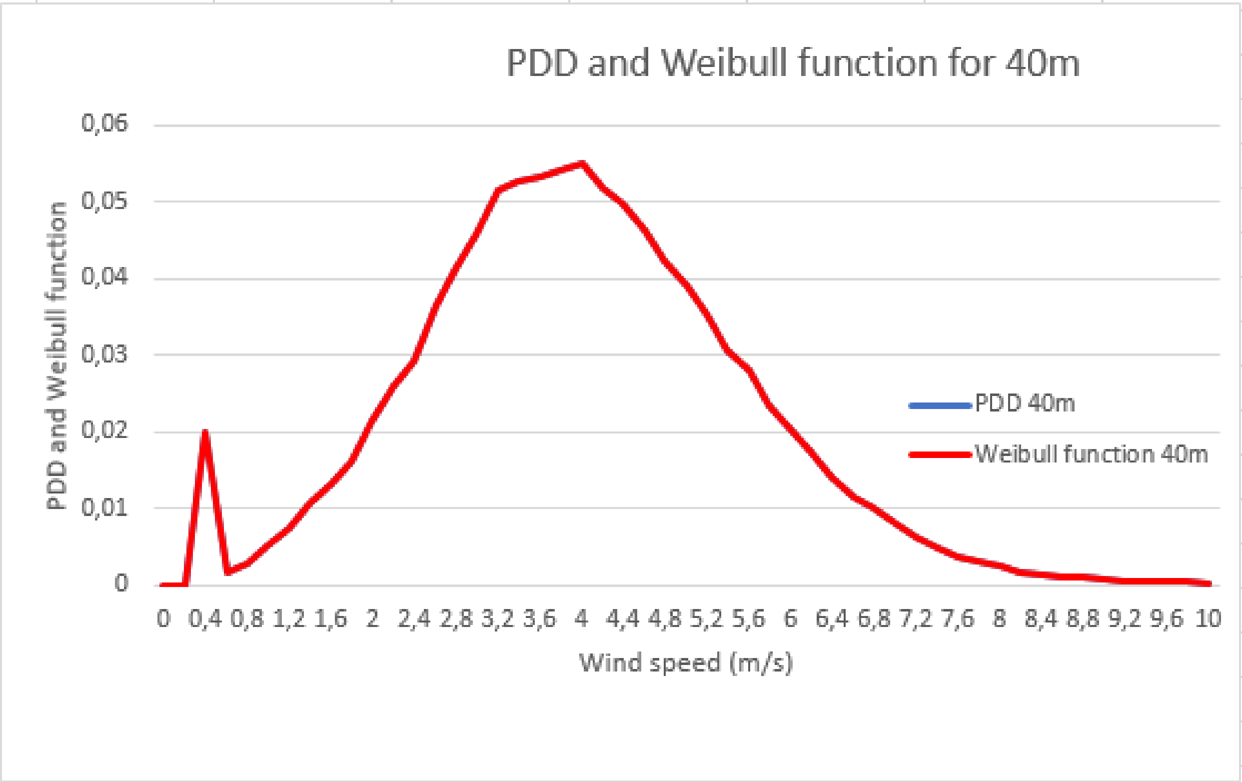

Figure 4.6: Probability Density Distribution (PDD) and Weibull function at 40m

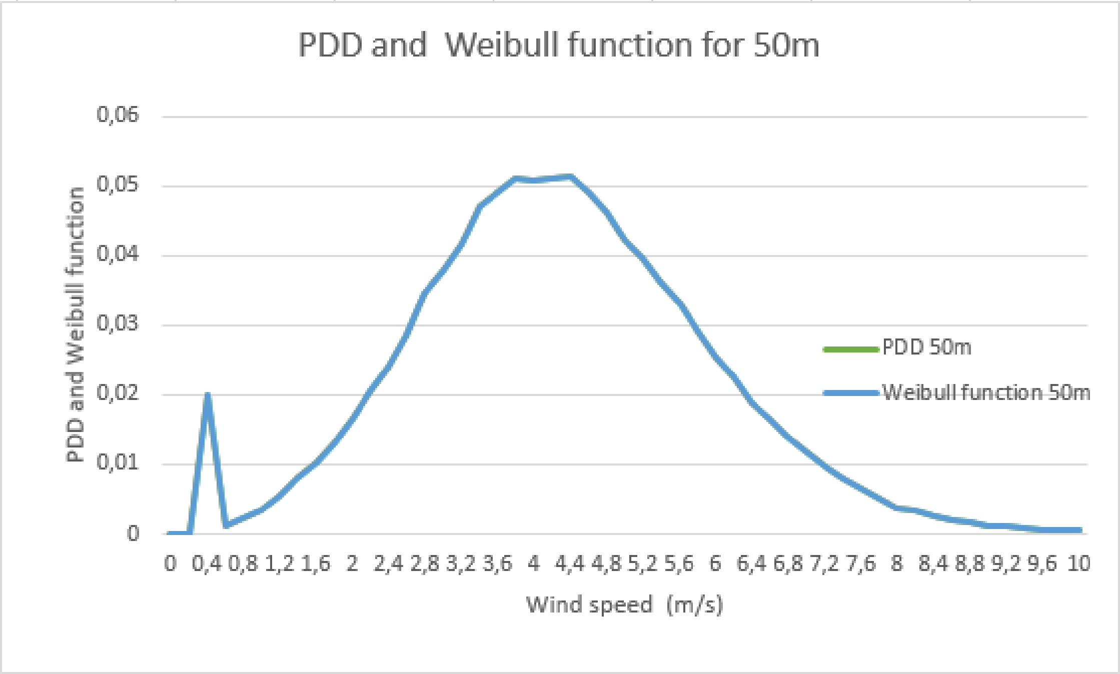

Figure 4.7: Probability Density Distribution (PDD) and Weibull function at 50m

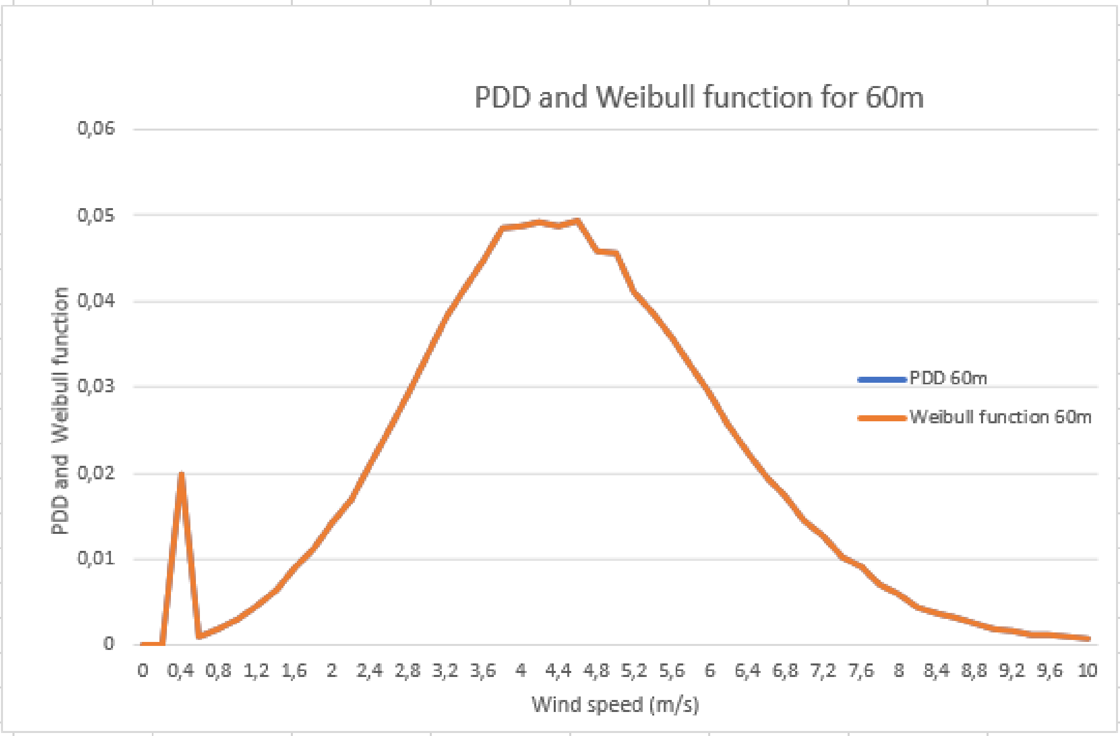

Figure 4.8: Probability Density Distribution (PDD) and Weibull function at 60m

To provide the wind speed data at different heights we will use the data at 60m since the wind speeds measurement distribution are almost the same on the three heights.

Using wind shear average ($\overline{\alpha}$) and the power law, the wind speed data are provided from a minimum height of 20m to a maximum height of 120m increasing at intervals of 5m. At each wind speed data the standard deviation and average wind speed are calculated.

The Weibull functions $P(V)$ at each height are determined from the shape parameter k and the scale factor c also calculated at each height using Equations (3.2.3) and (3.2.4), and compared with Equations (4.3.1) and (4.3.2).

The values of different parameters calculated are given on the Tables 4.3, 4.4 and 4.5.

Height (m)	20	25	30	35	40	45	50
Average wind speed $\overline{V}$ (m/s)	3,181	3,399	3,588	3,756	3,907	4,046	4,175
Standard deviation (σ)	1,246	1,331	1,405	1,471	1,531	1,585	1,635
Shape parameter k	2,767	2,767	2,767	2,767	2,767	2,767	2,767
Scale factor $c\,(m/s)$	2,832	3,025	3,193	3,373	3,478	3,601	3,716

Table 4.3: The averages wind speed, standard deviations and Weibull parameters

Height (m)	55	60	65	70	75	80	85
Average wind speed $\overline{V}$ (m/s)	4,294	4,407	4,513	4,613	4,708	4,799	4,886
Standard deviation (σ)	1,682	1,726	1,768	1,807	1,844	1,880	1,914
Shape parameter k	2,767	2,767	2,767	2,767	2,767	2,767	2,767
Scale factor $c\,(m/s)$	3,822	3,922	4,016	4,106	4,191	4,272	4,349

Table 4.4: The averages wind speed, standard deviations and Weibull parameters

Height (m)	90	95	100	105	110	115	120
Average wind speed $\overline{V}$ (m/s)	4,970	5,050	5,128	5,202	5,275	5,345	5,412
Standard deviation (σ)	1,947	1,978	2,009	2,038	2.066	2,094	2,120
Shape parameter k	2,767	2,767	2,767	2,767	2,767	2,767	2,767
Scale factor $c\,(m/s)$	4,423	4,495	4,564	4,630	4,695	4,757	4,817

Table 4.5: The averages wind speed, standard deviations and Weibull parameters

The power output defined in Equation (3.3.5), is calculated using two different wind turbines. In this Equation, the principle is to compute the new power coefficient $C_p(V_i)$ where V_i is given in Equation (3.3.3) by interpolation at each wind speed point for these two turbines. The power coeffecients are presented in Figures 4.3 and 4.5. The number *n* of divisions is 25 for Enercon E-18 turbine and 18 for Gaia-wind turbine.

5. Results and Discussion

The objective of this research project is to optimise the hub height of wind turbine which will improve the power output using a wind speed resource. This essay is focused on the wind speed data at 60m furnish in Kafue Gorge. The model used Gaia-wind and Enercon E-18 wind turbines. The rated powers of these turbines are 80kW and 11kW respectively. All the calculations were done using Excel software and the Python programming language.

The results obtained shows that the power generated by Enercon E-18 is far much greater than the power of Gaia-wind. This means that the power generated is proportional to the capacity of the wind turbine. Tables 5.1 and 5.2 give the different power output from 20m to 120m.

Height (m)	20	25	30	35	40	45	50	55	60	65
Enercon E-18 (kW)	1.025	1.351	1.675	2.065	2.316	2.635	2.948	3.262	3.572	3.878
Gaia-wind (kW)	0.058	0.123	0.200	0.305	0.377	0.473	0.570	0.669	0.769	0.868

Table 5.1: The power generated at different heights

Height (m)	70	75	80	85	90	95	100	105	110	115	120
Enercon E-18 (kW)	4.182	4.484	4.784	5.083	5.376	5.666	5.956	6.246	6.530	6.811	7.094
Gaia-wind (kW)	0.967	1.065	1.163	1.260	1.354	1.448	1.541	1.633	1.722	1.811	1.899

Table 5.2: The power generated at different heights

It is observed in Figures 5.1 and 5.2 that the power produced in both turbines is largest at the highest height, that is at 120m. These results obtained are a particular case, they are due to the fact that the wind resource used in this essay is of low speed.

This type of wind resource requires turbines with a lower power than those used in this project, in order to obtain the cut-out speed at lower heights.

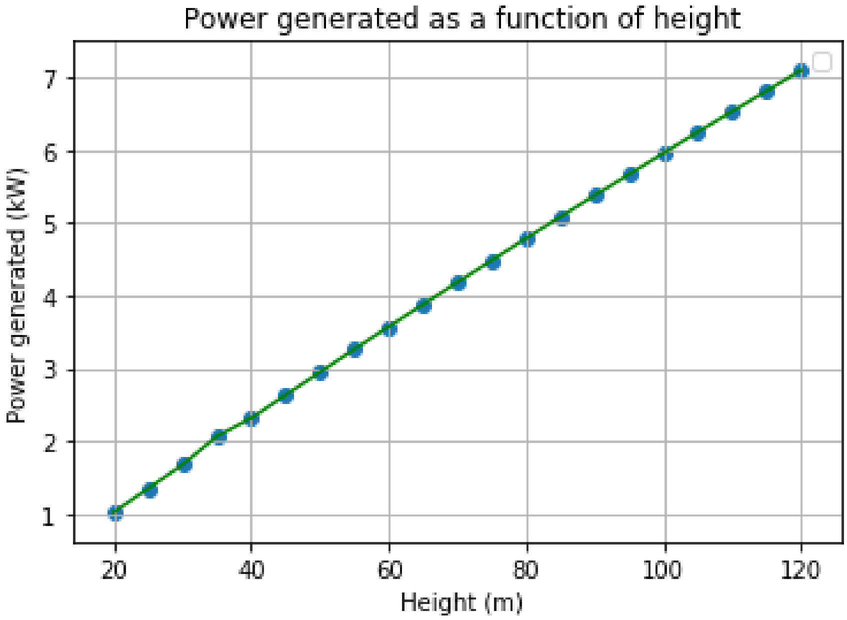

Figure 5.1: Power generated by Enercon E-18 wind turbine

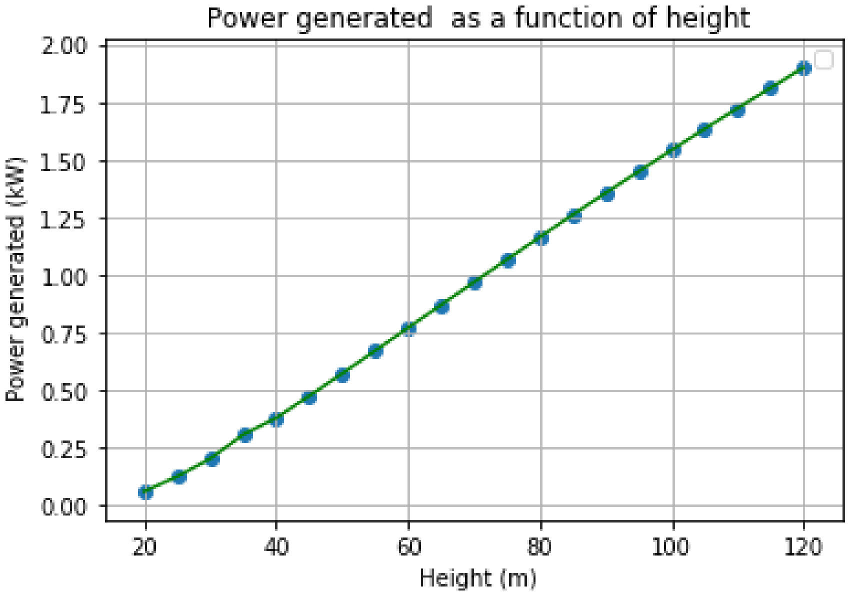

Figure 5.2: Power generated by Gaia-wind turbine

In theory, the power should reach a maximum at a given height and decrease thereafter, due to the cut-out speed of the wind turbine. Figure 5.3 shows the theoretical power produced by a turbine as a function of the height.

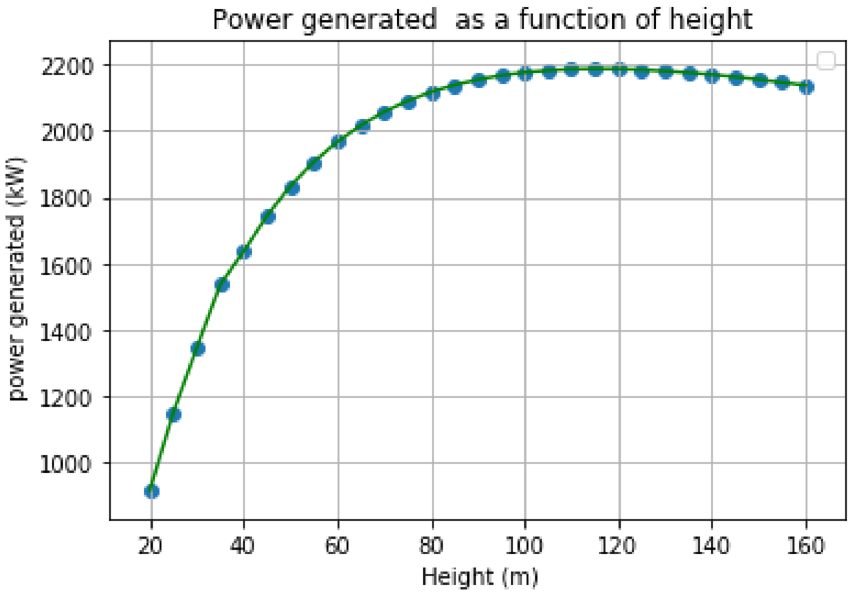

Figure 5.3: Theoretical power generated by a wind turbine

Finally, Figures 5.4 and 5.5, show the power coefficient of Enercon E-18 and Gaia-wind turbines and the probability density function at heights of 50m, 100m and 120m. A region of overlap is observed between the power coefficient and the probability density function (Weibull function) in the two cases of wind turbines. This shows the amount of power produced. In the interval above 10m/s, the Weibull function has the same value for these three heights.

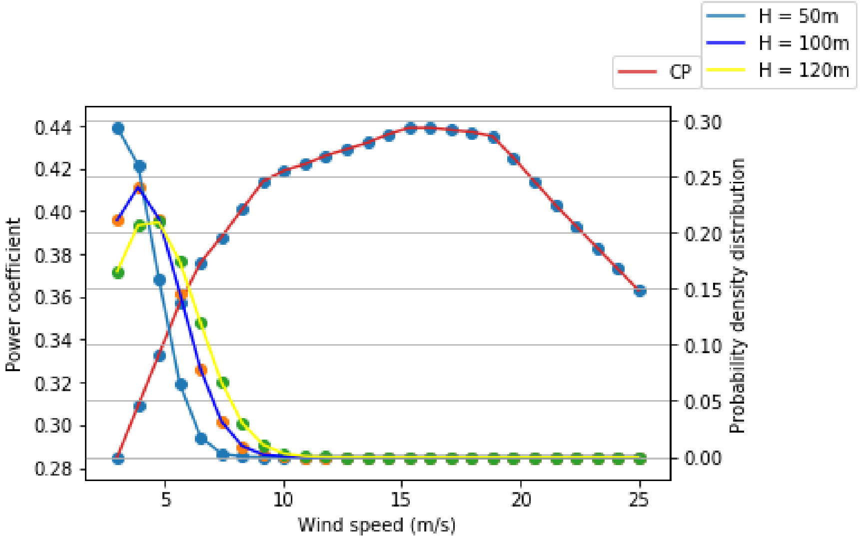

Figure 5.4: Probability density distribution and power coefficient for Enercon E-18 wind turbine

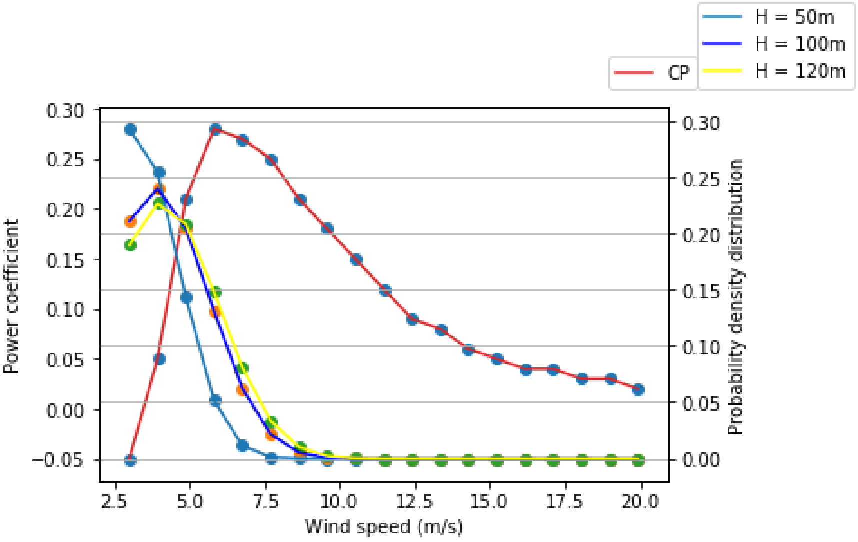

Figure 5.5: Probability density distribution and power coefficient for Gaia-wind turbine

6. Conclusion

The power output of a wind turbine is very variable due to wind speed fluctuations. It depends on many parameters including the selection of a wind turbine and an optimal hub height. In this research project, we investigated the optimal hub height for the power output of a wind turbine.

Firstly, we established the mathematical model of the power must be generated by a wind turbine. The power generation equation is solved using a numerical method known as the rectangles method. To obtain the data at different heights we used the wind speeds data at height $60m$. The new data was obtained from a height of $20m$ to a height of $120m$ increasing at intervals of $5m$. We calculated the Weibull function at each height from shape parameter k and scale factor c both obtained at the different heights. Secondly, we computed the power generated over different heights using two wind turbines, the Gaia-wind and the Enercon E-18 whose rated power were $11kW$ and $80kW$ respectively.

The results showed that the maximum power generated in both cases occurred at the highest height. These results are particularly for the wind speed data provided at Kafue Gorge, which is very low, and the rated power of the wind turbines used is high. In the case that the wind resource had a high wind speed or the turbines used had a low rated power, the maximum will be reached with a lower height.

For future work, we recommend solving the equation of the power generated by a wind turbine using a classical method and comparing this to a numerical solution. In the cases of a low wind resource, used likewise a wind turbine with low capacity, in order to get the optimal power at the lower height.

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