Valuation of Credit Default Swaps with Haskell

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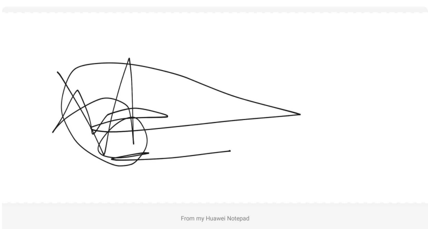
Abstract

Credit default swaps are known to be the most traded credit derivatives on the market whose main purpose include speculation, arbitrage and hedging. From their inception in the 90's there have been disparities of their trading in the market. CDSs have been priced by structural and reduced form models. In this essay we focus on the reduced form model with no counterparty risk that uses a discounted cash flow method for pricing CDSs. Our aim is to use Haskell, a functional programming language known for its key features. We implement the valuation of CDSs for the each specification under the reduced form model in Haskell.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.



Veronica Mtonga, 23 May 2019

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1. Introduction

Credit risk is a major risk that have on impact on many institutions (Gökgöz et al., 2014). Due to losses that may arise against this risk, many institutions use Credit default swaps as a hedging strategy against risk. According to Cont et al. (2010), CDSs are among the most traded Credit derivatives that are traded over the counter. Apart from hedging of credit risks, CDSs are also used for speculation and arbitrage purposes. CDSs came into existence because of the call to reduce losses that came about due to credit risk. Thus the idea lies in knowing the probability of credit event occuring. Hence the approaches that evaluate this risk are directly related to calculating the probability of default. At the inception of CDSs, banks were the prominent players in the market because they mainly used them to hedge the credit risk from the borrowers[1] but this changed in the 2002's where other players such as investors as speculators, as opposed to banks as hedgers where influential in the market. We look into the role that the CDSs played in the 2007-2008 financial crisis.

The uplifting of limits on leverage for banks in the united states and unregulated trading of CDSs in the financial sector especially in the year 2000 was disadvantageous for the economy before the financial crisis. As stated in[1] red flash lights of financial instability began showing in early 2000, by 2008 big investments banks like, Lehman brothers went bankrupt with an outstanding debt of $400 million. Later in that year, the American International Group (AIG), one of the biggest insurance banks and the seller of credit default swaps sold CDSs to several investors as AIG then, was highly rated by Mood's. AIG was one of the banks with the highest leverage ratio of 33:1, it had no insurance for counter-party risk. When Goldman Sachs, one of the investments banks that bought the CDSs from AIG defaulted and things went wrong, as the bubble burst AIG suffered $100 billion losses. Despite it been bailed out, the losses that incurred were too immense. Furthermore the outstanding on CDS's balance by the end of 2007 was $62.2 trillion and failed to $ 26.3 trillion by early 2012 (Gretarsson and Ennab, 2009).

Meanwhile, Stulz (2010) argued that the 2007-2008 financial crisis was not trigged by any financial derivatives. He claimed that neither Bear Sterns nor Lehman failed because of derivatives. Instead suggested two other factors that caused the financial crisis.

Firstly, prior to the crisis, a lot of unregulated sub-prime mortgage lending was evident. The investors and financial institutions had no expectation of the dramatically falling prices in that period. This fall was what was seen to have ignited large defaults on sub-prime mortgages and down falls in the value of securitisations of sub-prime mortgages with an outstanding balance that amounted to $ 421 billion in 2006 alone with estimations from Standard and Poor's. Secondly, several financial institutions that had large investments in sub-prime mortgage also operated with high levels of leverage. For example, investments banks in the united states in this period, despite being rated highly by Mood's, had unreasonable high leverage ratio. When the bubble burst, there was call for bail out for most of the banks. The losses that where encountered brought damage to the financial sector. After these great incidences as mentioned in Aldasoro and Ehlers (2018) the experience called for transparency and resilience. The need to regulate the trading of credit derivatives came in play and now it is observed that, the market of CDSs is slowly rising in comparison to the early 2000's with much regulation.

As stated in Gökgöz et al. (2014) and Choudhry (2012), the two main models for valuation of CDSs are structural and reduced models. In general, the three approaches in pricing CDS as mentioned in Gökgöz et al. (2014) are hedge-based valuation, bond yield valuation and discounted cash flow method. These

[1] Adopted from http://en.wikipedia.org/w/index.php?title=Credit%20default%20swap&oldid=892953657, last accessed 30th April 2019.

models use different assumptions and each have a particular way in which they price CDSs. The reduced model is best known for its no-arbitrage principle while the structural model is known for its pricing of CDSs by solving for the chance of a credit event associated with the value of the firms financial assets Gökgöz et al. (2014).

CDS are among the most important credit derivatives in the financial world. As the CDSs market keeps on growing despite the major down fall during the financial crisis it is important to find an easier or a friendly language that can be used to calculate or find a fair price for CDSs. Thus in this essay we seek to use Haskell's key features as a tool to price CDSs with functions that are continent for the pricing procedure.

This is how the essay is structured; in Chapter 1 we look into the market of CDS, then in the literature review we discuss the CDSs pricing models. We will present definitions and notations used in this essay in Chapter 2. Chapter 3 will consists of the key parameters in pricing CDSs and the three approaches under the reduced from models and how they price CDSs. Finally we present our findings of coding in Haskell in Chapter 4.

1.1 Literature Review

Credit default swaps are a subset of credit derivatives that are highly traded and have revolutionised the trading of credit risk. We discuss the initiation of CDSs by referring to the write up in [1] as follows: To the best of our knowledge, CDSs were first heard of in the late 90s. They were invented by JP Morgan in 1994. In simple terms, a credit default swap works in a similar way like ordinary insurance but the difference is that, unlike insurance where the one seeking insurance needs a reference obligation, with a CDS, one can buy a credit default swap without holding a loan instrument and not have a direct insurable interest in a loan. Over the years the market of CDSs has changed.The volume of Credit Default swaps traded increased around 2003. As stated in Gretarsson and Ennab (2009) and Gökgöz et al. (2014) the global credit derivatives market was worth \$180 billion in 1997. Which later increased in size to $\$350$ billion in 1998, \$ 586 billion in 1999, \$ 893 in 2000 and \$1,189 billion end of 2000. That is how fast the CDSs market was greatly moving.

Credit default swaps, as stated in Schönbucher (2000) despite the importance they have, little work on the pricing process has taken place. But generally as mentioned in Moore (2003) in Europe, in the early days, the pricing of these derivatives varied significantly from one provider to another. As discussed in Moore (2003), pricing of CDSs is more of an art than a science. But in recent years, we observe that CDSs pricing has become more standardised. In spite of this, the price of protection is not the same among participants in the market. It varies and there is a platform for shopping around for the best sellers of protection. However, all the sellers of protection use similar parameters in pricing CDSs. There are a number of detailed CDSs pricing models that range from structural, transitional to reduced from (Moore, 2003).

1.1.1 Structural Models. Also known as firms based models as discussed in Jarrow and Protter (2004) and Gökgöz et al. (2014) originated from Black and Scholes and Merton models that use the option pricing method. These models characterises the default of the firm as a consequence of some event that causes the firm's asset insufficient to repay the outstanding debt. The probability of default as stated in Choudhry (2012) is calculated by using the Black-Scholes-Merton option pricing theory. The

[1] Adopted from http://en.wikipedia.org/w/index.php?title=Credit%20default%20swap&oldid=892953657, last accessed 30th April 2019

Merton model in particular is the earliest of these models and is said to be the basis of credit risk models (Gökgöz et al., 2014). This model is said to be limited as it assumes one zero coupon bond. We describe extensions of the Merton model to fit economy problems in the following as discussed in Gökgöz et al. (2014).

Firstly, the Black and Cox model considers a single extension of the Merton's model by including a feature of calculating early default as a first time-passage. While Geske as stated in Karrlind and Tancred (2005) modelled defaultable coupon debt as a compound option on the firm's value, where defaults are triggered when the value is insufficient to meet the coupon obligations . Finally Longstaff and Schwartz models loosened the Mertons constant interest rate assumption and introduced a stochastic interest rate in the Merton's model. In-spite of the extensions on the Merton's model, some modellers as stated in Choudhry (2011) argue that, the Merton's model is still convenient for pricing default swaps on high-yield bonds. Furthermore, these models require the firm's balance sheet to establish a link between pricing in the equity and debt market. However, the limitations lie in; its difficulty to calibrate because the internal firms data is rarely published in the year. Secondly, it is too rigid to fit the exact given term structure of spreads. Furthermore, the parameters needed for the the value of the firms assets to be attained are not directly observed. Lastly, this model cannot be easily extended to price credit derivatives. (Moore, 2003).

1.1.2 Reduced form Models. Because of the limitations of the structural models made according to Duffie and Singleton (1999) reduced models were formulated and these models originate from Jarrow and Turnbull model and are a form of no arbitrage models that can be fitted to the term structure of risky-bonds to generate no-arbitrage prices Choudhry (2012). In these models, the probability of default is treated as a separate entity from the firm's asset. Instead, the probability of default is calculated for, by directly modelling it. The key reduced models that give detailed modelling of the probability of default are, Jarrow and Turnbull model whose focus is on modelling default and credit migration. It uses a risk-neutral transition matrix that is a key feature in the pricing process. But this model depends on historical transition matrix whose application does not fit future patterns. The Das and Tufano's model is an extension of Jarrow and Turnbull model that adopts the same risk-neutral transition matrix but with additional feature of stochastic recovery rates. The Duffie and Singleton model considers three types of risks namely, risk-free rate, hazard rate and recovery rate.

The question is, are there structural models better than the reduced models? There has been debate in the academic and professional literature as to which of these models is the best. But Moore (2003) mentioned in his paper that this debate usually evolves around default prediction and hedging performances. Jarrow and Protter (2004) also argued that the two models are the same and that there differences lies in the assumptions they have. The argument is that the structural models can be extended to price like reduced form models by refining the information from the observable firm's management to that observable by the market.

1.1.3 A Brief on Haskell. Haskell is a general purpose, purely functional programming language with strong static typing. Haskell programs are usually short lines of codes compared to other imperative languages like Python. In addition, the language has the following interesting features as highlighted in Hutton (2016)

- **<u>Shorter lines of code</u>**

  The algorithms are functional in nature, precise and concise. A simple example is summing elements in a list, i.e.

```
-- Sum of a list of n integers
```

```
summa n = sum [i | i <- [0,1..n]]
```

The function summa takes an argument n which is a number of integers whose sum you want to calculate.

- **Type annotations**

  Everything in Haskell has a type and this guarantees some sort of type safety especially when dealing with large sophisticated code. Moreover, the code won't compile when the types are incorrect which makes debugging pretty simple and fast. This feature is very powerful especially in systems where security is crucial such as in financial systems. Using the above example, we introduce the type annotations as follows:

```
-- This function takes an integer and returns an integer
summa :: Integer -> Integer
summa n = sum [i | i <- [0,1..n]]
```

  summa is a function that takes an integer and outputs an integer.

- **Tail recursion**

  Unlike most imperative languages that use loops to traverse lists, Haskell employs tail recursion for similar operations. This results in shorter and readable lines of code. This concept utilizes what is termed as "Pattern Matching". Below is an illustration:

```
-- Pattern matching and tail recursion
factorial :: Integer -> Integer
factorial 0 = 1
factorial x = x * factorial (x-1)
```

  The equivalents for *if* statements in other languages is achieved through what is termed as Guards, below is an example:

```
--Guards
factorial2 :: Integer -> Integer
factorial2 x | x == 0 = 1
| otherwise = x * factorial2 (x-1)
```

  Guards are very useful as they help one test weather a function is true or false.

- **Higher-Order functions**

  In Haskell, functions can be used as arguments to other functions. This is equivalent to functions of functions in mathematics. See an illustration in the following example that calculates the difference between two numbers of a list.

```
--this is a function that takes in an integer and gives outputs its f
factors :: Int -> [Int]
factors n = [ y | y <- [1..n-1], n`mod` y == 0]

--we define a function that returns perfect numbers
perfect :: Int -> [Int]
perfect z = [t | t <- [1..z] , sum (factors t) == t]
```

- **Monadic programming and side effects control**

  High level and complex Haskell algorithms are built on Monads which are blocks of code designed to execute sequences of commands and where variables within a monad are not affected by those outside the monad.

- **Lazy evaluation**

  The idea is that Haskell won't evaluate or compute any operation until such a point when its result has to be used. For instance the assigned $x = 100 * 50$ will not be evaluated until such a point when x has to be printed, used by another function or returned. Only a pointer or reference with be linked to x. This is a great future for both speed and good memory usage.

  For a simple and quick introduction to Haskell, the reader is advised to visit the online resource at Learn You a Haskell for Great Good!.

2. A General Overview on CDSs

2.1 Definitions

This section presents definitions, terminologies and notations used in this essay.

2.1.1 Definitions.

2.1.2 Basis Point. Is a unit of measurement used in finance for the description of the percentage change in the value or rate of a financial instrument. 5 basis point is equivalent to 0.05% or $(5/100^{th})$ of a percent. An example is, 10 basis points is equivalent to 0.1% [1].

2.1.3 Bootstrapping. This is a test or metric that depends on random sampling with replacement. It makes it possible for the assignment of measures of accuracy to sample estimates [2].

2.1.4 Bond. This is a fixed income security that represents a loan given by an investor to a borrower, that can be a corporation or a government. Bonds issued by the government are sovereign and are regarded risk-free compared to the bonds issued by a corporation Hull (2003).

2.1.5 Coupon. This refers to interest payments made on a bond. It is the price a bond issuer pays for issuing a bond Hull (2003).

2.1.6 Credit Event. A credit event is legally an event that includes bankruptcy, failure to pay or defaulting and restructuring. The most discussed form of a credit event is defaulting (Karrlind and Tancred, 2005).

2.1.7 Credit Swap Spread. This refers to the periodic payments of a credit default swap made by the protection buyer to the protection seller to maturity or until a credit event occurs. This is usually quoted in basis points per annum of the contract's face value, which is sometimes referred to as credit swap premium (O'Kane and Turnbull, 2003).

2.1.8 Credit Rating. This refers to the credit worthiness of a bond issue. It is a focus into the credibility of paying back a debt or the likelihood of a particular debtor defaulting.

2.1.9 Hazard Rate. This involves the measuring of the probability of default respectively or the survival probability (Schmidt, 2016). It can also be defined as the rate or probability of an event that is expected to occur over a given period of time given that an event has not yet occurred.

2.1.10 Leverage Ratio. This is a financial ratio that represents the level of debt sustained by a business entity against several other accounts in its balance sheet, income statement or cash flow statement.

2.1.11 Liquidity. This refers to how easily assets can be moved into cash without losing their value.

2.1.12 Maturity Date. When an investor makes an investment or a lender gives out credit they do so over a period of time. The last date or repayment date is what is called the maturity of the investment or the debt respectively. It is basically the end of life of a financial instrument.

2.1.13 Poisson Process. GOKGOZ (2012) This is a non-decreasing process N_t with intensity $\lambda > 0$ satisfying the following conditions:

1. Initial value $N_0 = 0$

[1]Adopted from, http://en.wikipedia.org/w/index.php?title=Plagiarism&oldid=895111181, Accessed on 3rd May, 2019
[2]Adopted from,https://en.wikipedia.org/wiki/Bootstrapping_(statistics), on 20^{th} May, 2019

2. The process has non dependent and stationary increments.
3. The density function of the process has the form $\mathcal{P}(N_t = n) = \frac{(\lambda t)^n}{n!} \exp(-\lambda t)$
4. For $0 < s < t$, the random increment $N_t - N_s$ has the poisson distribution with parameter $\lambda(t-s)$ and

$$\mathcal{P}(N_t - N_s = n) = \frac{\lambda^n (t-s)^n}{n!} \exp(-\lambda(t-s)).$$

2.1.14 Protection Leg. The protection fee sometimes referred to as the contingent fee refers to the amount that the protection seller makes to the protection buyer in an event of a default. This fee is only paid upon the occurrence of a credit event. Usually the protection seller is expected to pay after 72 calender days. But it differs from the parties involved in the agreement (O'Kane and Turnbull, 2003).

2.1.15 Premium Leg. The premium leg consists of two parts: Regular premium (or coupon) payments (e.g. every three months) up to the expiry of the CDS, which ceases if a default occurs; and a single payment of accrued premium in the event of a default. The latter is not always included in all CDS contracts (O'Kane and Turnbull, 2003).

2.1.16 Recovery Rate. This is the amount recovered in an event of a default expressed as a percent of the face value (Hull, 2012). It is usually expressed in percentage.

2.1.17 Reference Entity. This is a company as defined in Hull (2003) that issues a bond. It can either be a corporate or a sovereign institution. An example could be the borrower of a loan or a bond.

2.1.18 Risk-less Investment. This is a type of investment in which the return is known with certainty, an example is a treasury securities are known to be risk-less as the government is known to be the best possible issuer [3].

2.1.19 Risky Investment. This is an investments in which the return is not guaranteed for sure. The securities issuers are likely to default, go bankrupt or fail to repay back what they owed [3].

2.1.20 Yield. This is the return that is provided by an instrument. An example is when an investor or lender who invests or lends money respectively. The earnings or profits that they expect to receive is what is the yield or return on their investment or debt (Hull, 2003).

2.1.21 Zero Coupon Bond. This is a type of bond without coupon payments. A zero coupon bond can be either corporate or sovereign (Hull, 2003).

2.2 Notations

Symbol	Meaning
S_c	Contractual spread.
d_n	Discount factor from time t to t_n.
N	Total number of coupons.
T	life of the credit default swap in years.
q(t)	Risk-neutral probability density at time t.
R	Expected recovery rate on the reference obligation in a risk-neutral world.
u(t)	Present value of payments at a rate of $\$1$ per year on payment dates between time zero and time t.
e(t)	Present value of an accrual payment at time t equal to $t - t^*$ where t^* is the payment date immediately preceding time t.
v(t)	Present value of $\$1$ received at time t.
ω	Total payment per year made by credit default swap buyer.
s	Value of ω that causes the value of the credit default swap to have a value of zero.
π	Risk-neutral probability of no credit event during the life of the swap.
A(t)	Accrued interest on the reference obligation at time t as a percentage of the face value.
s_N	Par spread(CDS premium) maturity N.
R	Recovery rate of the reference obligation.
t_m	Time to maturity in years
$\wedge$	And

Table 2.1: All notations used in this essay

2.3 Credit Default Swap

According to Gretarsson and Ennab (2009), a CDS is a credit derivative instrument that is a bilateral contract between a protection buyer and a protection seller. It is used to transfer credit risk of a reference obligation up to some fixed date. This reference obligation can be a loan or a bond. It basically involves three parties namely: the bond issuer, the lender or the protection buyer and the protection seller. The protection buyer decides to be protected from a credit event that will arise from the bond issuer by buying protection from the protection seller for a stipulated period of time. The buyer makes regular payments to the protection seller till a credit event occurs or to maturity, which depends on what occurs first. These regular payments can either be fixed or floating depending on the agreement between them. When a credit event occurs the protection seller then pays the protection fee to the protection buyer. And the protection buyer pays the accrued payments between the previous premium payment and the time of the credit event.

2.3.1 Example. We refer to the example given in O'Kane and Turnbull (2003).

Given that, Company A purchases a 5 year maturity CDS from Company B with reference $\$5$ million notional value of a bond to company C with contractual spread of 250bp. Company A makes payments to Company B in a quarterly frequency. In an event that, default happens, we assume company C has a 35$\%$ recovery rate. To price this CDS, we consider two possible cash flows:-

Cash flow 1 If default has not occurred then, the cash flows are as follows:

After every quarter, the protection buyer will pay $\$5$ million $\times$ 0.025 $\times$ 0.25 $= \$31,250$. Thus, the protection buyer makes total premium payments of $\$625,000$ million to the protection seller until maturity.

Meanwhile, Company B makes no payments to Company A because there is no occurrence of default.

Cash flow 2 Assume Company C defaults a month in the 3rd year after the previous premium payment then the protection buyer pays the accrual payments $\$5$ million $\times$ 0.025 $\times$ $1/12 \approx \$10,417$ and the protection seller, an amount of $\$5$ million $\times$ (100% - 35%) $= \$3.25$ million.

Figures 2.1 and 2.2 give an illustration of the above CDS contract.

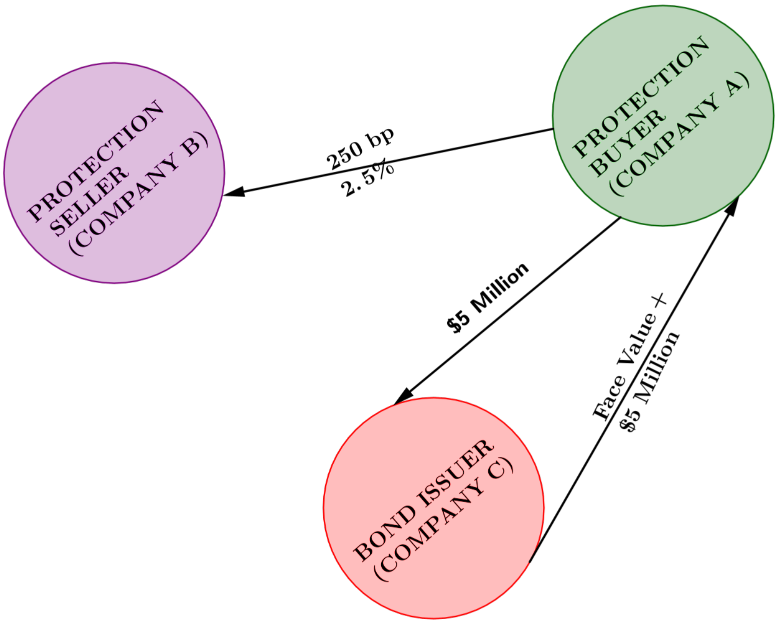

Figure 2.1: Illustration of a Credit default swap with no default dynamics.

Figure 2.2 shows a CDS with an occurrence of default.

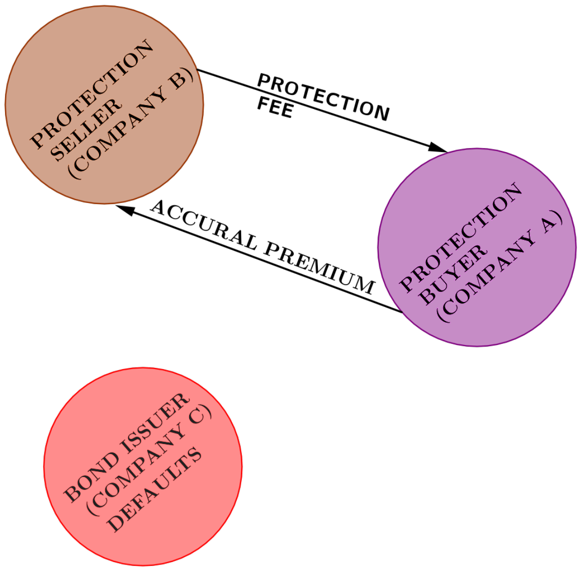

Figure 2.2: Cash flow in a CDS when default occurs.

The basic idea behind pricing a CDS lies in finding a fair price. This means a price without arbitrage. It therefore requires the two cash flows in both legs to be equal. Before we look into the models for pricing CDS we first discuss the parameters that play a key role in CDSs pricing processing.

2.4 Essential Parameters in CDS Valuation

From the example 2.3.1, we notice vital parameters required in valuing a CDS and these include:

1. Probability of default.
2. Timing of default.
3. Maturity of a CDS.
4. Recovery rate.

We briefly discuss these parameters in the following.

2.4.1 Default Probability. Determining the probability of default is not always a straight froward task. Nevertheless, various literatures (for instance Choudhry (2012)) have shown these probabilities could be extracted from the credit spreads of the corporate bond market. risk-less investments, like the government bond establish are used as a proxy for the risk-less interest rate. Corporate bonds are preferred over their government bond counterparts because they offer a higher yield as a result of the high risk component they carry. The difference between the yields from the risk-less rates and the risky rates is termed as credit spread and depends on the following:

- Credit quality
- Liquidity
- Maturity

- Supply and demand

Another way of calculating probability of default is by using the hazard rate which we discuss in detail under the standard approach in Chapter 3.

2.4.2 Time of Default. The time of default is random and is modelled using a Poisson process introduced above.

2.4.3 Recovery Rate. The recovery rate is another important parameter that can either be deterministic or random. The recovery rate that we use in this essay for most of the approaches is a constant recovery rate that we be based on the data that we shall use.

2.4.4 Risky and Risk-less Rates. A risk premium is the return in excess of the risk-free rate of return an investment is expected to yield; an asset's risk premium is a form of compensation for investors who tolerate the extra risk, compared to that of a risk-free asset, in a given investment[3].

[3]See Investopedia.

3. Pricing CDSs

There are several specificationes used for pricing CDSs that fall under the reduced form models framework and we discuss three in this essay namely; standard, theoretical and market specification.

3.1 Market Specification

This specification is based on the no-arbitrage principle, see Duffie (1999) and Choudhry (2012), and it is what the market uses. Moreover, the model assumes the following:

1. no arbitrage principle.
2. a constant recovery rate.
3. a given term structure of interest rates.
4. an actual /360-day count convention.

The probability of default under the market specification is computed according to the following.

3.1.1 Estimation of Default Probability. Consider a continuously compounded rate of return on the risk-free asset given by e^{rt} with r, t as the risk free yield and maturity respectively, and the rate of return of the risky asset given by $e^{(r+y)t}$ where y is the risky spread on the investment. Assuming a zero recovery rate and a probability of default denoted by p, an investor should be indifferent when the two returns are equal. That is

$$\boxed{p = 1 - e^{-yt}}. \tag{3.1.1}$$

In the presence of a recovery rate R, an investor is indifferent under

$$(1-p)e^{(r+y)t} + Rpe^{(r+y)t}. \tag{3.1.2}$$

By equating the Expression (3.1.2) to e^{rt} we have

$$\boxed{p = \frac{1 - e^{-yt}}{1 - R}}.$$

Let $\mathcal{D}$ denote default event with a probability q and s represent survival with probability of $(1-q)$. This is represented as a binary process in Figure 3.1.

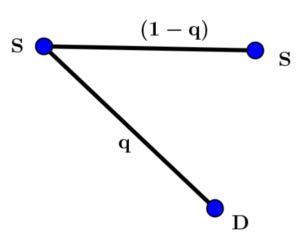

Figure 3.1: Binary process of survival and default events

Over a several periods this binary process follows the structure in Figure 3.2.

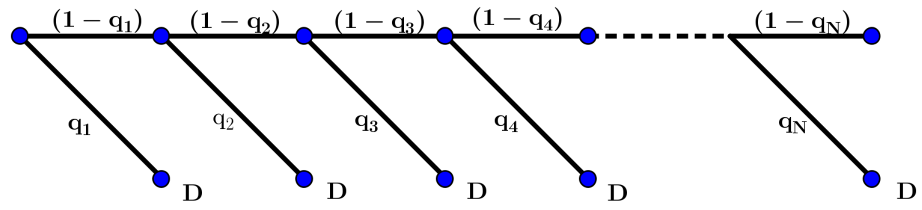

Figure 3.2: Representation of a binary process of survival or default over several periods

From Figure 3.2 the probability of survival to the Nth period denoted by (PSN) is given by

$$PSN = (1 - q_1) \times (1 - q_2) \times (1 - q_3) \times (1 - q_4) \times \cdots \times (1 - q_N). \quad (3.1.3)$$

The probability of default in any period denoted by (PDN) is

$$PSN + 1 \times PDN = PSN + 1 - PSN. \quad (3.1.4)$$

Consequently, the price of a CDS is derived in the following.

3.1.2 Pricing. Given a set of default probabilities, we consider a series of contractual spreads (s_n[1]) and cash flows contingent on whether or not a credit event occurs. See Figures 3.3 and 3.4 respectively.

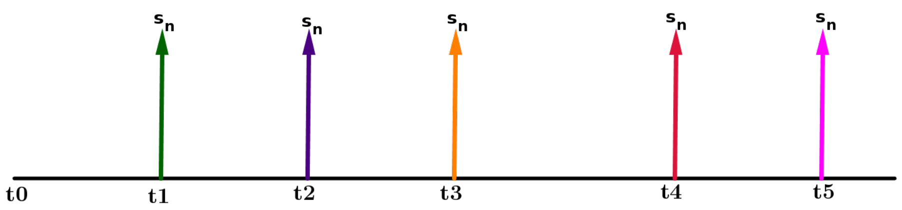

Figure 3.3: Description of the price of a CDS without default, s_n is the CDS premium with t_i is the period.

[1]This refers to the fixed size of a regular coupon

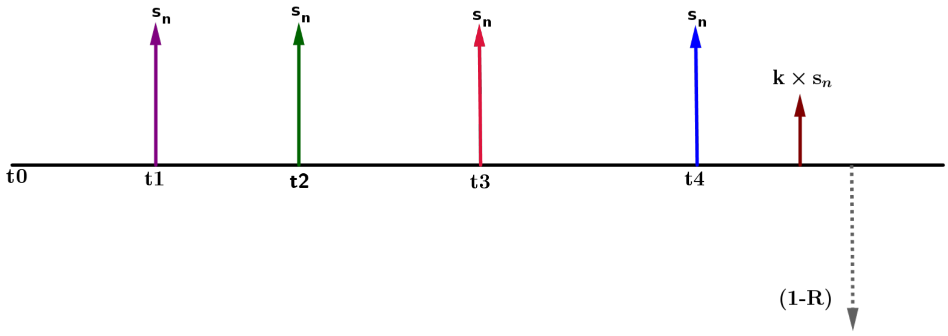

Figure 3.4: CDS contingent cash flow when default occurs, s_n is the CDS premium.

From Figure 3.3 and for no default, the present value of streams of CDS premiums (PVS$_{nd}$) over time is calculated as

$$PVS_{nd} = s_n \sum_{j=1}^{N} (DF)_j (PS)_j T_{j-1,j}. \tag{3.1.5}$$

Where,

$(DF)_j$, $(PS)_j$ are discount factor and probability of survival in any j period respectively. $T_{j-1,j}$ length of time of period j.

The premium streams given a default through period C as shown in Figure 3.4 yields

$$PVS_d = s_n \sum_{j=1}^{C} (DF)_j (PS)_j T_{j-1,j} + s \times (DF)_C (PD)_C \left(\frac{T_{C-1,C}}{2}\right), \tag{3.1.6}$$

where,

$(PD)_C$, $T_{C-1,C}$ is the probability of default to period C and the length of time to period C respectively. $(DF)_C$ is the discount factor to period C.

With the default payment given by

$$(1 - R)(DF)_C (PD)_C. \tag{3.1.7}$$

For no-arbitrage (3.1.6) and (3.1.7) are equal with a possible occurrence of default in any period j. This yields a fair premium of a CDS given by

$$s_n = \frac{(1 - R) \sum_{j=1}^{N} (DF)_j (PD)_j}{\sum_{j=1}^{N} (DF)_j (PS)_j \left(T_{j-1,j}\right) + \sum_{j=1}^{N} (DF)_j (PD)_j \left(\frac{T_{j-1,j}}{2}\right)}. \tag{3.1.8}$$

3.1.3 Example Calculation. Choudhry (2012).

Consider a 1 year CDS and 6 month CDS. Using the parameters in Table 4.3, the price of CDS is computed as follows. From (3.1.5), we have for a 1-year period with s_1 the CDS spread given by

$$0.9983 \times 0.9643 \times s_1 \times 1.0 = 0.96266069 s_1.$$

For the 6-month period with s_2 as the CDS spread is

$$0.9993 \times 0.9826 \times s_2 \times 0.5 = 0.49096 s_2.$$

The present value if default happens half way through the period using the second part of the R.H.S of Equation (3.1.6) is given, for a 1-year period as

$$0.0017 \times 0.9643 \times s_1 \times 0.5 = 8.19655 \times 10^{-4} s_1.$$

For the 6 month-period we have

$$0.0007 \times 0.9826 \times s_2 \times 0.25 = 1.7196 \times 10^{-4} s_2.$$

Thus the premium leg for a 1 year and 6 month period is

$0.96266069 s_1 + 8.19655 \times 10^{-4} s_1 = 0.9634803 s_1$ and $0.49096 s_2 + 1.7196 \times 10^{-4} s_2 = 0.491132 s_2$

The present-value of the default (contingent) payment, if payment is made at the end of the period is obtain from (3.1.7) and for a 1 year period, we have

$$0.0017 \times 0.9643 \times (1 - 30\%) = 1.1475 \times 10^{-3}. \quad (3.1.9)$$

For a 6-month period we have

$$0.0007 \times 0.98266 \times (1 - 30\%) = 4.815 \times 10^{-4}. \quad (3.1.10)$$

Therefore, the CDS price for a 1 year and 6 months is given as

$$\begin{aligned} 0.48221 s_1 &= 1.1475 \times 10^{-3} \\ s_1 &= 0.002362 \end{aligned} \qquad \begin{aligned} 0.491132 s_2 &= 4.815 \times 10^{-4} \\ s_2 &= 0.000989 \\ &= 0.0010 \end{aligned}$$

Thus for the 1 year period, the price of the CDS is 0.236% and 0.1% for the 6-month CDS.

3.2 Standard Pricing Specification

We refer to the works of Schmidt (2016), Choudhry (2012) and O'Kane and Turnbull (2003) in the following. The reduced form model consists of two cash flows, the premium leg and the protection leg. The model assumes the following:

(i) a credit event always occurs in the middle of two coupon payments.

(ii) the probability of default structure is known.

(iii) under the protection leg, the pay-off occurs immediately after a credit event to simplify matters.

(iv) a constant recovery rate.

This specification uses a hazard rate to calculate the probability of default as discussed in the following.

3.2.1 Probability of default from the hazard term structure. The probability of default is given by:

$$\text{Probability of default } (P) = 1 - \text{Survival probability}. \tag{3.2.1}$$

A credit event is modelled as a Poisson counting process (see O'Kane and Turnbull (2003) and Schmidt (2016). The probability of a credit event in a time interval $[t,\ t+dt)$ under the condition that there has not been a default until t is as follows:

$$P(\tau < t + dt | \tau \geq t) = \lambda(t)dt. \tag{3.2.2}$$

We denote by $\lambda(t)$ the hazard term structure or the hazard rate. As a result, the conditional survival probability $Q(t,T)$ until time T given that t has been reached is given by

$$Q(t,T) = \exp\bigg(- \int_t^T \lambda(s)ds \bigg). \tag{3.2.3}$$

With no forward CDS evaluation, Equation (3.2.3) reduces to:

$$SR(T) = Q(0,T) = \exp\bigg(- \int_0^T \lambda(s)ds \bigg). \tag{3.2.4}$$

Equation (3.2.4) is called the survival probability denoted by $(SR(T))$ until time T.

The hazard rate can take on various properties or forms depending on the model. Historical default rates leads to default probabilities. Alternatively, they can be extracted directly from market prices of CDS and in turn obtain the hazard rate or hazard term structure.

3.2.2 Constant Hazard Rate. Let $\lambda = \lambda(t)\ \forall t \in [0,T]$, then

$$\begin{aligned} SR(t) &= \exp\bigg(- \int_0^t \lambda ds \bigg), \\ &= \exp(-\lambda t). \end{aligned}$$

Making the following assumptions:

- λ is always non-negative.
- λ is defined in the domain $0 < \lambda < 1$.

A possible approximation of the implied λ is calculated using both the Bisection and Netwon method described in O'Kane and Turnbull (2003). The authors priced the CDS using the contractual spread and the implied hazard rate. The method is simple and fast but leads to default probabilities that, are not the same when different maturities are used.

Figure 3.5 shows the hazard rates defined by $\lambda = \lambda(t)$. Notice that the longer the maturities, the higher the hazard rates.

The probability of default with reference to the constant hazard rate is given as

$$\begin{aligned} P &= 1 - SR(t), \\ &= \exp(-\lambda t). \end{aligned} \tag{3.2.5}$$

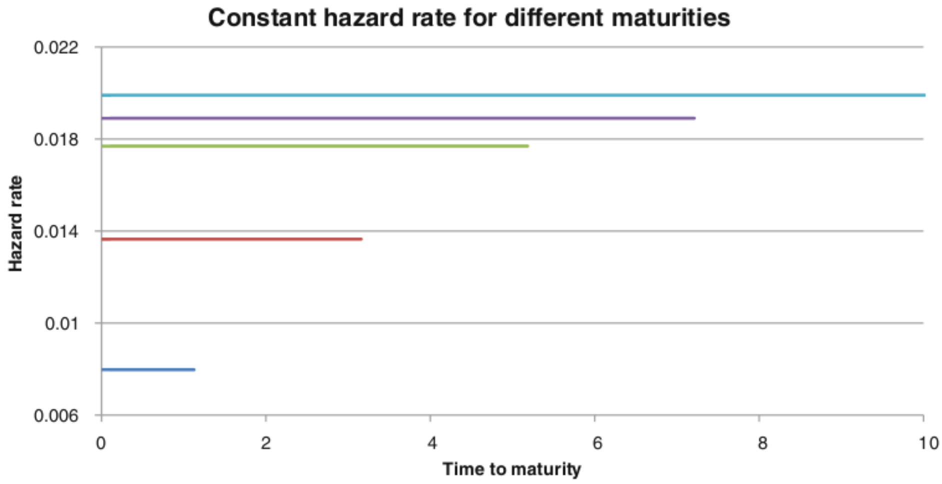

Figure 3.5: Hazard rates under the constant hazard rate term structure with CDS market data from Badische Anilin-Und Soda-Fabrik (BASF) (Schmidt, 2016)

Figure 3.6 shows the probability of default defined by (3.2.5).

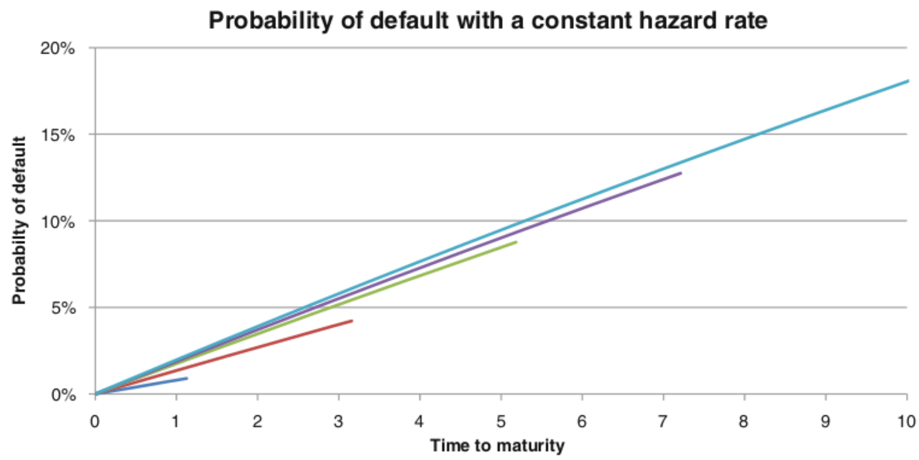

Figure 3.6: Probability of default versus the constant hazard rate with market data of CDS from BASF (Schmidt, 2016)

From Figure 3.6 a particular hazard rate, gives a different probability of default for the same period. Thus, under a constant hazard rate, we have results of implied probability of default that are not the same.

3.2.3 Partial Constant Hazard Rate. Market quotes from maturities that are not the same are used to bootstrap for a unique hazard rate that is described in O'Kane and Turnbull (2003). It uses a hazard rate built as follows:

$$\lambda(t) = \sum_{i=1}^{L} \lambda_i. \tag{3.2.6}$$

given that, $L = \min\{i|1 \leq K \wedge t_i \geq t\}$ with survival rate as

$$SR(t) = \exp\Big(-t\sum_{i=1}^{L}\lambda_i\Big).$$

The parameters λi are obtained from the market quotes with a method of bootstrapping as described. This model uses market data which is implied and it has one probability of default curve (Schmidt, 2016). In-spite of this, the model requires more data which is not easily accessible, the method employed gives out results that take longer computing time for maturities that are long. lastly the quality of data maybe different between maturities.

The probability of default is defined by

$$\begin{aligned} P &= 1 - SR(t) \\ &= 1 - exp\Big(-t\sum_{i=1}^{L}\lambda_i\Big) \end{aligned} \tag{3.2.7}$$

Figure (3.7) below shows the partial constant hazard rates as defined by equation (3.2.6) and the obtained probability of default from (3.2.7).

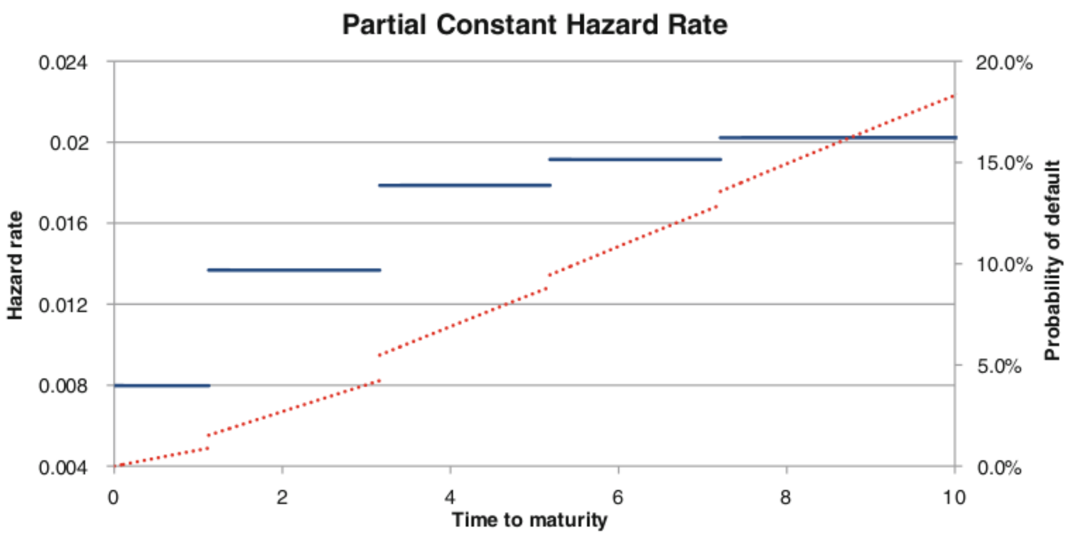

Figure 3.7: The hazard term structure and the probability of default from the CDS market data from BASF (Schmidt, 2016)

The blue and red dotted lines represents the hazard term structure and the probability of default respectively. From Figure (3.7) we observe that, the probability of default is unique because one hazard rate exists for each period.

3.2.4 Linear Hazard Rate. To get a more realistic probability curve, we use the linear hazard rate. In this case the hazard rate term structure is modelled via:

$$\lambda(t) = \lambda t. \tag{3.2.8}$$

With a survival rate of:

$$SR(t) = \exp(-0.5\lambda t^2).$$

The probability of default is given by:

$$\begin{aligned} P &= 1 - SR(t), \\ &= 1 - \exp(-0.5\lambda t^2). \end{aligned} \tag{3.2.9}$$

With the same derivation of the parameters as discussed in Schmidt (2016) under the constant hazard rate. Figure (3.8) shows the probability of default defined by Equation (3.2.9) using the linear hazard rate.

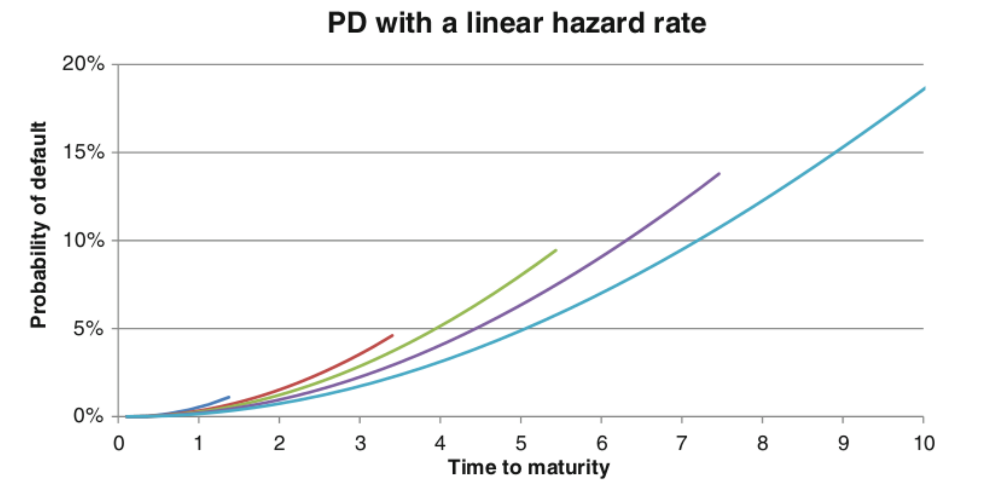

Figure 3.8: The probability of default from the CDS market data for BASF (Schmidt, 2016)

From Figure (3.8) we observe that different hazard rates are obtained that lead to probabilities of default that are not the same. We note that, the probability of default decreases for longer maturities which is the opposite case of what was observed under the constant hazard rate.

3.2.5 Partial linear Hazard Rate. This specificationes uses a similar method as the one described under the partial constant hazard rate above of finding λ but with the assumption that the hazard rate is constructed by this formula :

$$\lambda(t) = \sum_{i=1}^{L} \lambda_i t.$$

with a survival rate of

$$SR(t) = \exp\Big(-0.5t^2 \sum_{i=1}^{L} \lambda_i \Big).$$

With a probability of default defined as:

$$\begin{aligned} P &= 1 - SR(t) \\ &= 1 - \exp\Big(-0.5t^2 \sum_{i=1}^{L} \lambda_i \Big). \end{aligned} \tag{3.2.10}$$

The hazard rate is always different at two time points that are not the same, and with the assumption $\lambda i > 0$ the hazard rate is a monotone increasing function. Figure (3.9) shows the probability of default defined by (3.2.10).

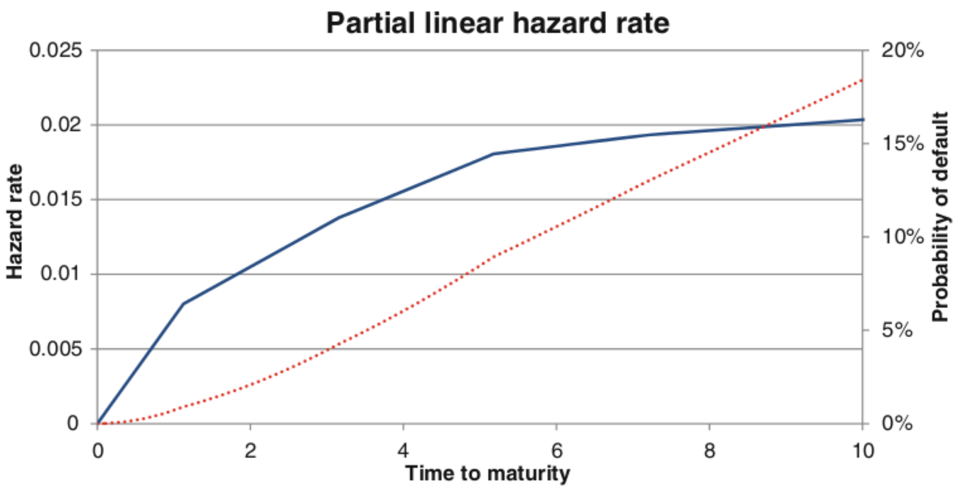

Figure 3.9: The hazard term structure with a partial linear hazard rate and the corresponding default probability from the CDS market data for BASF (Schmidt, 2016)

As shown in Figure (3.9) the blue line represents the partial linear hazard rate and the probability of default in red. It shows that, there is only one probability of default curve.

3.2.6 Pricing CDS under Standard Specification. According to Schmidt (2016), the pricing of the premium leg and protection leg are described as follows:

$$\text{premium leg} = \frac{S_c}{2} \sum_{n=1}^{N} \Delta(t_n - t_{n-1}) d_n \Big(Q(t, t_{n-1}) - Q(t, t_n) \Big). \tag{3.2.11}$$

Pricing the protection leg is dependant on the time of a default and recovery rate. Given the assumption that the timing of a credit event every year occurs on a finite points that are discrete. We can then price the protection leg as follows

$$\text{Protection leg } = (1-R) \sum_{i=1}^{M \times t_m} d_i (Q(t, t_{i-1}) - Q(t, t_i)). \tag{3.2.12}$$

The no-arbitrage theory requires that the premium and protection leg be equal. Thus from (3.2.11) and (3.2.12), we price a CDS as follows:

$$S_c = \frac{(1-R) \displaystyle\sum_{i=1}^{M \times t_m} d_i (Q(t, t_{i-1}) - Q(t, t_i))}{\frac{1}{2} \displaystyle\sum_{n=1}^{N} \Delta(t_n - t_{n-1}) d_n (Q(t, t_{n-1}) - Q(t, t_n))}. \tag{3.2.13}$$

Equation (3.2.13) represents the formula for calculating a CDS under the standard specification. We now look at the theoretical specification of pricing CDSs.

3.3 Theoretical Specification

Similar to the standard specification the default swap is also valued like an interest swap that is described in Choudhry (2012). The Hull and White (2000) model falls under this specification. As stated in this model, the present value of the of a CDS is valued as the present value of the algebraic summation of the two legs. The model has the following assumptions:

1. no counter-party default risk.
2. default probability, recovery rates and interest rates are not dependant on each other.
3. in an event of default, the claim is the face value with addition of the interest accrued. The calculation of probability of default under this specification is discussed in the next section.

3.3.1 Calculating Probability of default. We refer to the method of calculating the probability of default discussed in Gretarsson and Ennab (2009) that uses a hazard rate term structure contrary to using the default density concept. In this essay we use the method that was described in Gretarsson and Ennab (2009) as follows:

The method used is that of the hazard rate as described under the standard specification above due to the fact the construction of a zero-curve for all the 20 companies that were described by using the bootstrapping method as described in (O'Kane and Turnbull, 2003) with some adjustments to the hazard rate construction process. Thus we can now calculate the price of the CDS under the theoretical specification as follows:

3.3.2 Pricing a CDS under theoretical Specification. This model according to Wen and Kinsella (2013) is used to price a plain vanilla $\$1$ notional value CDS and is calculated as follows:

As described in Choudhry (2012) and Hull (2003) the value of π which represents the survival probability is calculated as follows:

The meaning of the notations below are described in Section 2.2

$$\pi = 1 - \sum_{i=1}^{n} q_i. \tag{3.3.1}$$

In the event of no default then, the present value of the CDS payments is $\omega u(T)$. But when default takes place at $t(t < T)$, then the present value of the payment is $\omega[u(t) + e(t)]$. Thus the present value of the premium payment is:

$$w \sum_{i=1}^{n} q_i \Big[u(t_i) + e(t_i) \Big] + \omega\pi u(T). \tag{3.3.2}$$

With the assumption of the claim amount, the risk neutral expected pay-off of the CDS contract is derived as:

$$1 - R[1 + A(t_i)] = 1 - R - A(t_i)R.$$

Now the present value of the expected pay-off of the CDS is given by:

$$\sum_{i=1}^{n} \Big[1 - R - A(t_i)R \Big] q_i v(t_i). \tag{3.3.3}$$

As stated in Hull and White (2003) taking Equation (3.3.2) = (3.3.3) we have that the present value of the expected pay-off is equal to the present value of the value of the expected payments. Thus taking ω we equate (3.3.2) and (3.3.3) then

$$\text{Credit Spread} = \frac{\sum_{i=1}^{n} \Big[1 - R - A(t_i)R\Big] q_i v(t_i)}{\sum_{i=1}^{n} q_i \Big[u(t_i) + e(t_i)\Big] + \pi u(T)}. \tag{3.3.4}$$

Equation (3.3.4) gives a general formula for calculating a CDS that allows default per year . For allowing default to occur at any time, taking $q(t)$ as the risk-neutral default probability density at time t. Equation (3.3.4) is now,

$$\text{Credit Spread} = \frac{\int_0^T \Big[1 - R - A(t)R\Big] q(t)v(t)dt}{\int_0^T q(t)\Big[u(t) + e(t)\Big] dt + \omega\pi u(T)}. \tag{3.3.5}$$

4. Results and Discussion

4.1 Results

In this chapter we present the findings of pricing credit default swaps in Haskell using the methods and models described in Chapter (2) and (3). All the results presented here are based on the Haskell code that we used and can be accessed from Git-Hub[1].

4.1.1 Data. The data shown in Table (4.1) is retrieved from Choudhry (2012) and we use it to calculate the default and survival probabilities as step one and will further calculate the price of CDSs.

Maturity t	Risk-Free Yield r($\%$)	Corporate Bond Yield r+y($\%$)	Risk Spread y($\%$)
0.50	3.57	3.67	0.10
1.00	3.70	3.82	0.12
1.50	3.81	3.94	0.13
2.00	3.95	4.10	0.15
2.50	4.06	4.22	0.16
3.00	4.16	4.32	0.16
3.50	4.24	4.44	0.20
4.00	4.33	4.53	0.20
4.50	4.42	4.64	0.22
5.00	4.45	4.67	0.22

Table 4.1: Hypothetical Corporate Bond Yields and Risk Spreads Choudhry (2012).

4.2 Market Specification in Haskell

4.2.1 Example. Given a recovery rate of 30% We first start with calculating the probability of default, survival probability and the discount factor using data from Choudhry (2012) as illustrated in Table 4.1. The results are presented in Tables 4.2 and 4.3.

[1] Access the code at, https://github.com/VickyVero/Haskell_Aims

Table 4.2: Calculated Probabilities of default with zero recovery rate

Maturity t A	Risk Spread(%) V	Survival Probability
0.500	1.000×10^{-3}	0.999
1.000	1.200×10^{-3}	0.998
1.500	1.300×10^{-3}	0.997
2.000	1.500×10^{-3}	0.996
2.500	1.600×10^{-3}	0.994
3.000	1.600×10^{-3}	0.993
3.500	2.000×10^{-3}	0.990
4.000	2.000×10^{-3}	0.989
4.500	2.200×10^{-3}	0.986
5.000	2.200×10^{-3}	0.984

Table 4.3: Calculated Probabilities of Default with a non zero Recovery rate

Maturity t A	Risk Spread(%) V	Default Probability
0.500	1.000×10^{-3}	7.141×10^{-4}
1.000	1.200×10^{-3}	1.713×10^{-3}
1.500	1.300×10^{-3}	2.783×10^{-3}
2.000	1.500×10^{-3}	4.279×10^{-3}
2.500	1.600×10^{-3}	5.703×10^{-3}
3.000	1.600×10^{-3}	6.841×10^{-3}
3.500	2.000×10^{-3}	9.965×10^{-3}
4.000	2.000×10^{-3}	1.138×10^{-2}
4.500	2.200×10^{-3}	1.407×10^{-2}
5.000	2.200×10^{-3}	1.563×10^{-2}

Secondly, Figure 4.1 and 4.2 below show the term structure of the probability of default and survival probability respectively. We observe that the probability of default of the reference entity increases with time. We see from Figure 4.1 and the opposite happens for the survival probability.

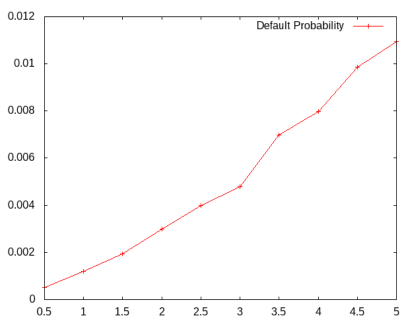

Figure 4.1: Term Structure Default of probabilities

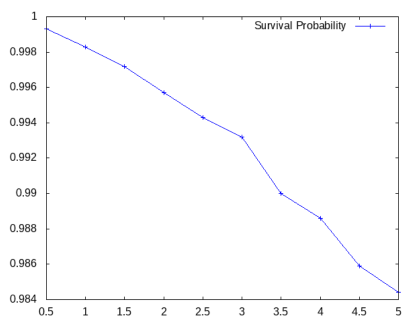

Figure 4.2: Term structure of Survival Probability

The CDSs for the data provided in Table 4.1 as calculated for using the market approach shown in Figure 4.3. We observe that the price of the CDS increases with an increase in maturity.

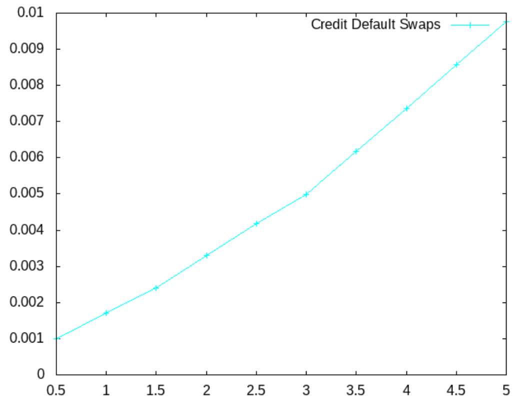

Figure 4.3: This is a Graph of Credit Default Spread as the y-axis with the Maturities as the x-axis

Figure (4.4) below shows the relationship between the credit default swaps spread and the probability of default. We observe that an increase in probability of default implies in increase in the price of CDSs. Thus the price of a CDSs is directly proportional to the probability of Default.

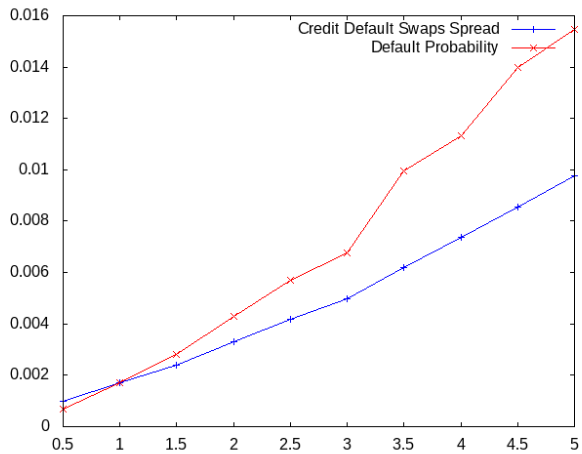

Figure 4.4: Relationship between the CDSs Spread and the Probability of Default

4.3 Theoretical Specification in Haskell

In this section we illustrate the results of pricing a CDS using the standard approach as discussed in Chapter (2).

4.3.1 Example of Pricing a CDS. we use an example that is used in Hull (2003) to price a CDS with the code as presented on the Git-Hub [1].

4.4 Discussions

The valuation of CDSs involves finding a fair price. For this, the two cash flows i.e from the protection buyer to the protection seller and the other way round must be the same. We note that, pricing of a CDSs depends on whether default occurs or not. Thus We observe that modelling the probability of default leads to different methods of pricing a CDS and thus under the reduced form framework we observed three specifications that have different ways of modelling the default probability.

The pricing of CDSs under the market and theoretical specifications are basically functions and it is for this reason that we chose Haskell for its powerful purely functional feature. The formulas are purely functional and in Haskell we are certain that a function called with the same parameters always returns the same result. This is very important in finance especially with the pricing procedure under the mentioned specifications. We are guaranteed that there are no side effects in Haskell that we alter the values under the pricing process. The functions defined in Haskell under the pricing specifications are as written under each pricing method for the specifications.

5. Conclusion

Credit derivatives are the leading trading credit derivatives with an increase in the common use. The reduced form model is preferred to the structural model when pricing CDSs due to the discussed advantages they have of fitting market observable parameters. In this essay, we discussed the market, theoretical and standard specifications that constitute the reduced form model for their ability to model the probability of default directly. We described the key parameters used in pricing CDSs under each specification. The three specifications discussed in this essay differed by the the methods of modelling the probability of default, the assumptions they had and there different specifications for calculating key parameters. For example, the standard specification unlike the theoretical and market specification employed the use of four hazard rates in the calculation of the probability of default.

We implemented the methods of pricing the CDSs under the market and theoretical specifications in Haskell. We are guaranteed of getting the needed results for the arguments of the pricing methods under each specification. With Haskell's lazy evaluation we are only use the functions only when needed. Finally the pricing of a CDS in Haskell is a great contribution to Haskell's libraries. We are confident that anyone who may stumble on a program in Haskell for pricing CDSs will be the one we just implemented in this essay.

5.0.1 Future Work. For future work, we recommend to create a Haskell engine for all the three specifications. Furthermore create a detail analysis and comparison of Haskell pricing engine to other engines in other programming languages. In addition, there is need to test the pricing engine on real market data. Finally, we wish to extend this valuation, to other credit derivatives or even other financial instruments options or stocks in Haskell.

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