Autocorrelation of an ultrashort laser pulse to determine its duration

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Abstract

Various techniques to find the duration of an ultrashort laser pulse have been developed. This work is a report of an experiment to determine the duration of an ultrashort laser pulse with 770 nm wavelength in both the temporal and spectral domain using autocorrelation technique. Autocorrelation technique is a technique based on the interaction of a pulse with a time delayed copy of itself within a medium. In this experiment I will exploit the phenomenon of second harmonic generation (SHG) and amplification of ultrashort laser pulse. Using measured values from the plots I will produce from the autocorrelation trace and spectrum respectively I will subsequently determine the pulse duration and spectra width in both frequency and wavelength. Lastly I will compute the spectral bandwidth product and determine if the pulse is transform limited or not.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Tamba William Saa, 22 May 2014

Contents

1. Introduction

Atomic nuclei vibrates around their equilibrium positions, binding electrons move into outer orbitals, reactants form new compound due to the arrangement of atomic nuclei and electrons. All of these processes takes place within only a few femtoseconds $(10^{-}15)$, fractions of a second so small like comparing one second to the age of the universe. This temporal window is experimentally accessible by ultrafast laser spectroscopy, which utilizes laser pulses of durations down to 10 fs to start photoinduced dynamics in matter and to probe the evolution of the excited system at later times. However, temporal resolution of these experiments cannot be achieved by any electronic detector, but must be accomplished by purely optical techniques which in essence take laser induced snapshots of the dynamics at mechanically determined delayed times.

My goal in this experiment is to introduce the formation of ultrashort pulse, describe the autocorrelation operation, measure an ultrashort laser pulse widths in frequency and time using autocorrelation technique. In the first part of my work I will explain the theory of the time and frequency domains than use the Fourier transform method to measure the pulse duration.The second part of this work describes the experimental set up with detail diagrams at the Laser Research Center at Stellenbosch University where the experiment was conducted. Data of the autocorrelation out put signal was collected and plotted to determine the pulse duration and spectral bandwidth. These results were use to calculate the time bandwith product and compare to the theoretical value of a transform limited pulse.

2. Theory: The Ultrashort Laser Pulse

A laser is an optical source that emits coherent radiation at a specific wavelength. Ultrashort laser pulse is an optical pulse with duration of picoseconds or less. It is one billionth of one millionth of a second.

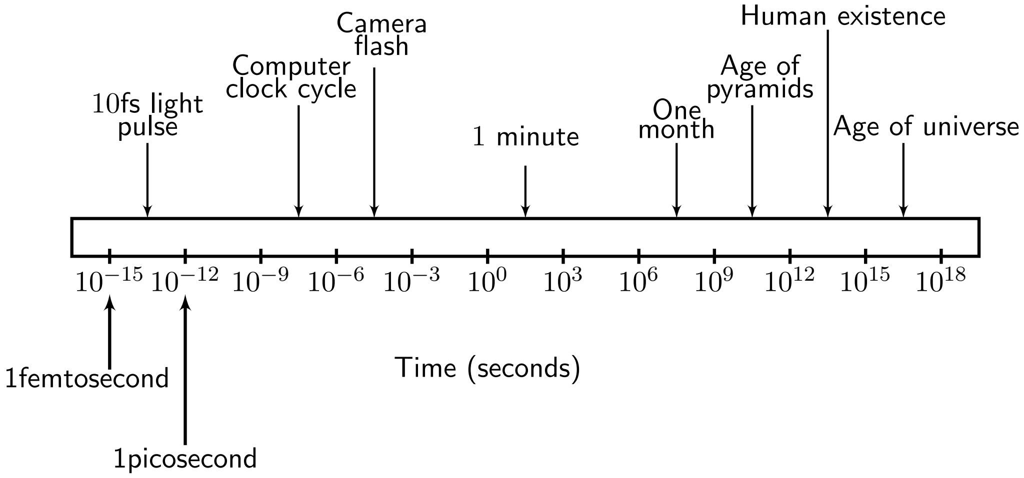

Figure 2.1: Time Scale

Femtosecond (fs) light pulses are electromagnetic wave packets and as such are fully described by the time and space dependent electric field.

A pulse is characterized by measurable quantities that can be directly related to the electric field. Consider the temporal dependence of the electric field neglecting its spatial and polarization dependence, i.e, $E(x, y, z, t) = E(t)$. We can completely describe a pulse in either the time or the frequency domain as we will do in the next section using Fourier transform to achieved this.

2.1 Time- and Frequency-Domain Measurements

The frequency domain is used to measure the spectrum and the spectral phase (phase of the electric field in the frequency domain) while in the Time-Domain we measure the intensity and the phase.

2.1.1 Time-Domain. In order to measure the duration of an event, you must use a shorter event. But than to measure the duration of a shorter event, you must use an even shorter one and so on. So, how do we measure the duration of an ultrashort laser pulse? We will use a pulse to measure itself. Now our problem becomes measuring a pulse by itself.

Consider the energy time uncertainty principle given by

$$\Delta E \Delta t \geq \frac{\hbar}{2}, \tag{2.1}$$

ΔE represents the standard deviation in energy and Δt is the amount of time it takes the expectation value of some operator to change by a standard deviation.

For photons we can write $E = \hbar\omega$ as

$$\Delta\omega\Delta t = \frac{\hbar}{2}. \quad (2.2)$$

It is obvious from equation (2.2) that Δt and $\Delta\omega$ are inversely related, a smaller Δt demands a larger $\Delta\omega$, or frequency range. This will be shown later by means of Fourier decomposition.

It is necessary therefore to use a laser with large gain bandwidth to generate ultrafast pulses. Titanium-doped sapphire ($Ti : Al_2O_3$) laser is used; it has gain bandwidth of 300 nm and over 10^6 modes in a typical laser cavity.

Because of the difficulty of asserting the exact pulse shape, standard waveforms have been selected. The most commonly cited is the Gaussian, for which the temporal dependence of the field is $E(t)$,

$$E(t) = \exp\left(-\frac{t^2}{\tau^2}\right)\cos(\omega_0 t), \quad (2.3)$$

which is plotted below using the values $\tau = 30$ fs , $\omega = 2\pi f$ where $f = \dfrac{c}{\lambda}$; $\lambda = 530$ nm is the pulse wavelength and c is the speed of light in vacuum. Hence, $\omega = \dfrac{2\pi c}{\lambda} = 3.554 \cdot 10^{15}$ Hz.

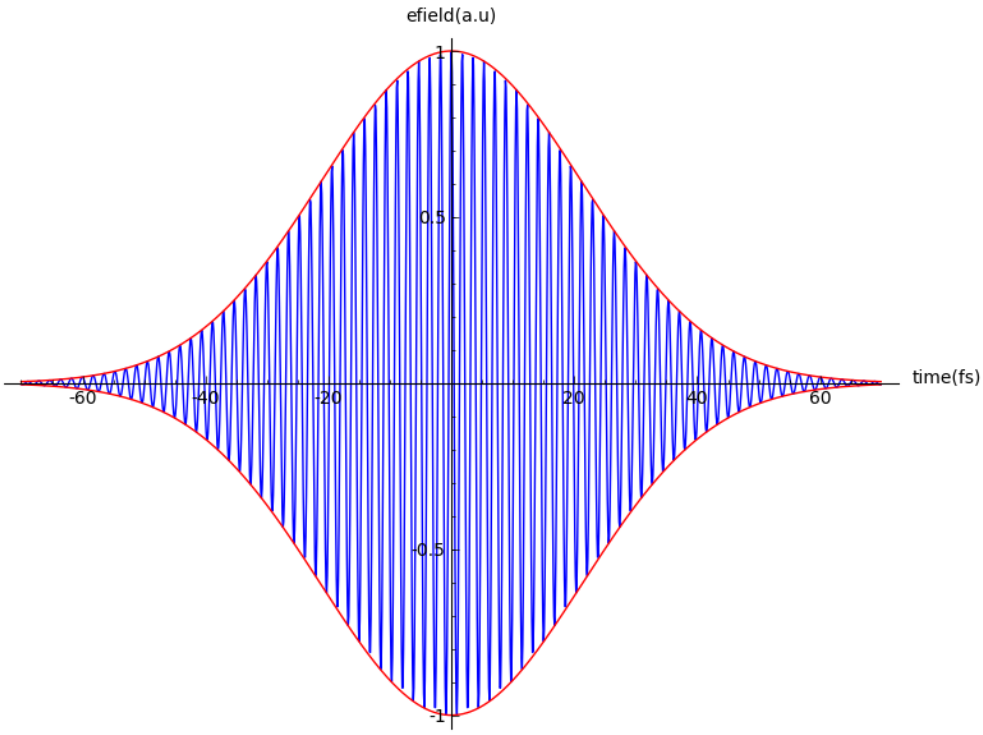

Figure 2.2: electric field intensity of an ultrashort laser pulse

2.2 The Frequency Domain

The frequency domain sometimes refers to as the domain of the spectrometer is generally used to measure the spectrum, $S(\omega)$, of a laser pulse.

The measurement of the duration τ of this laser pulse is achieved by finding the Fourier transform of the electric field $S(\omega)$ of the pulse.

2.2.1 Calculating the Spectrum. We will analytically calculate the Fourier transform of the spectrum of the Gaussian shape light pulse using Fourier transform technique.

The Fourier transform, is a mathematical transformation used to transform signals from time (or spatial) domain to frequency domain. It is reversible, being able to transform from either domain to the other. The term itself refers to both the transform operation and to the function it produces.

We therefore calculate the spectrum $S(\omega)$, from the electric field, $E(t)$, using the Fourier transform method.

$$E(\omega) = \int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \cos(\omega_0 t) \cdot \cos(\omega t) dt. \tag{2.4}$$

Recall the trigonometric identity

$$\cos(x) \cdot \cos(y) = \frac{1}{2}\{\cos(x-y) + \cos(x+y)\}, \tag{2.5}$$

therefore

$$E(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \left[\cos(\omega_0 - \omega)t + \cos(\omega_0 + \omega)t\right] dt,$$

$$E(\omega) = \frac{1}{2} \left[2 \int_{0}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \cos[(\omega_0 - \omega)t] dt + 2 \int_{0}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \cos[(\omega_0 + \omega)t] dt\right],$$

$$E(\omega) = \int_{0}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \cos[(\omega_0 - \omega)t] dt + \int_{0}^{\infty} \exp\left(-\frac{t^2}{\tau^2}\right) \cos[(\omega_0 + \omega)t] dt.$$

Now we expand and simplify the above expression using the Maclaurin series, a special case of Taylor expansion. By definition

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n}. \tag{2.6}$$

Therefore, for $x = (\omega_0 - \omega)t$ for the first integral and $x = (\omega_0 + \omega)t$ for the second integral we have

$$E(\omega) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (\omega_0 - \omega)^{2n} \int_{0}^{\infty} \exp\left(\frac{-t^2}{\tau^2}\right) t^{2n} dt + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (\omega_0 + \omega)^{2n} \int_{0}^{\infty} \exp\left(\frac{-t^2}{\tau^2}\right) t^{2n} dt.$$

Let

$$\frac{t}{\tau} = \sqrt{x}$$

$$t = \tau\sqrt{x},$$

$$dt = \frac{\tau}{2\sqrt{x}}dx.$$

Therefore

$$E(\omega) = \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 - \omega)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 + \omega)^{2n}\right] \int_0^{\infty} \exp(-x) \cdot (\tau\sqrt{x})^{2n} \cdot \frac{\tau}{2\sqrt{x}} \cdot dx,$$

$$E(\omega) = \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 - \omega)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 + \omega)^{2n}\right] \frac{\tau^{2n+1}}{2} \int_0^{\infty} x^{n-\frac{1}{2}} \cdot \exp(-x) \cdot dx.$$

From the definiton of the Gamma function, $\Gamma(n)$, we have

$$\Gamma(n) = \int_0^{\infty} x^{n-1} \exp(-x) dx, \tag{2.7}$$

similarly

$$\Gamma\left(n + \frac{1}{2}\right) = \int_0^{\infty} x^{n-\frac{1}{2}} \exp(-x) dx,$$

hence,

$$E(\omega) = \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 - \omega)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 + \omega)^{2n}\right] \frac{\tau^{2n+1}}{2} \cdot \Gamma\left(n + \frac{1}{2}\right).$$

By definition

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi} \cdot (2n)!}{4^n \cdot (n!)}. \tag{2.8}$$

Therefore,

$$E(\omega) = \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 - \omega)^{2n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(\omega_0 + \omega)^{2n}\right] \frac{\tau^{2n} \cdot \tau}{2} \frac{\sqrt{\pi} \cdot (2n)!}{4^n \cdot (n!)},$$

$$E(\omega) = \frac{\tau\sqrt{\pi}}{2} \left[\sum_{n=0}^{\infty} (-1)^n \left\{\left(\tau\frac{\omega_0 - \omega}{2}\right)^2\right\}^n \frac{1}{n!} + \sum_{n=0}^{\infty} (-1)^n \left\{\left(\tau\frac{\omega_0 + \omega}{2}\right)^2\right\}^n \frac{1}{n!}\right],$$

$$E(\omega) = \frac{\tau\sqrt{\pi}}{2} \left[\exp - \left(\frac{\tau(\omega_0 - \omega)}{2}\right)^2 + \exp - \left(\frac{\tau(\omega_0 + \omega)}{2}\right)^2\right].$$

For very large ω and ω_0 the second term vanishes. Hence our spectrum simplifies to

$$E(\omega) = \frac{\tau\sqrt{\pi}}{2} \cdot \exp - \left(\frac{\tau(\omega_0 - \omega)}{2}\right)^2. \tag{2.9}$$

where $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for $x = \left(\frac{\tau(\omega_0 \pm \omega)}{2}\right)^2$. ω_0 and ω are optical frequencies in the order of 10^{14th}, τ is the pulse duration assume to be longer than a few periods of the wave, being a few tens of femtoseconds.

2.2.2 Full Width at half Maximum. Consider a Gaussian

$$g(x) = C \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{2.10}$$

The maximum of this function occurs at

$$g(x) = C.$$

The half maximum of this function is therefore

$$\frac{1}{2} \cdot g(x) = \frac{1}{2} \cdot C.$$

If we let $h = x$ at half maximum we have

$$\frac{1}{2} \cdot C = C \cdot \exp\left(-\frac{h^2}{2\sigma^2}\right),$$

$$\frac{1}{2} = \exp\left(-\frac{h^2}{2\sigma^2}\right).$$

Taking natural log on both sides gives us

$$\ln\frac{1}{2} = \ln\left(\exp(-\frac{h^2}{2\sigma^2})\right),$$

$$\ln\frac{1}{2} = -\frac{h^2}{2\sigma^2},$$

$$-2\sigma^2\ln\frac{1}{2} = h^2,$$

$$-2\sigma^2(\ln 1 - \ln 2) = h^2,$$

$$-2\sigma^2(-\ln 2) = h^2$$

$$h = \sigma\sqrt{2\ln 2},$$

hence

$$\text{FWHM} = 2 \cdot \sigma \cdot \sqrt{2\ln 2} \tag{2.11}$$

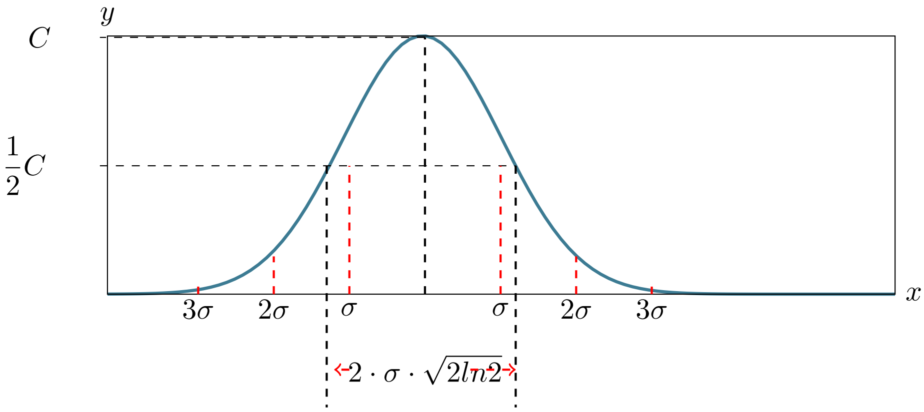

Figure 2.3: A gaussian plot showing the FWHM

2.2.3 Product of Pulse Duration and Spectral Width. If we compare equation 2.9 with the definition of a Gaussian P(x) $= \dfrac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$,

we have

$$\begin{aligned} \frac{1}{2\sigma^2} &= \frac{\tau^2}{4}, \\ \sigma^2 &= \frac{2}{\tau^2}, \\ \sigma &= \frac{\pm\sqrt{2}}{\tau}. \end{aligned} \tag{2.12}$$

Only the positive root is physically relevant here. Hence our spectral width is $\dfrac{2\sqrt{2}}{\tau}$.

Now our time-bandwidth product is

$$\Delta\omega \cdot \Delta\tau \geq k, \tag{2.13}$$

$$\Delta\omega \cdot \Delta\tau \geq 0.441, \tag{2.14}$$

where ω is the frequency bandwidth measured at full width at half maximum (FWM) and τ is FWHM in time and, k is a number which depends only on the pulse shape. Therefore,

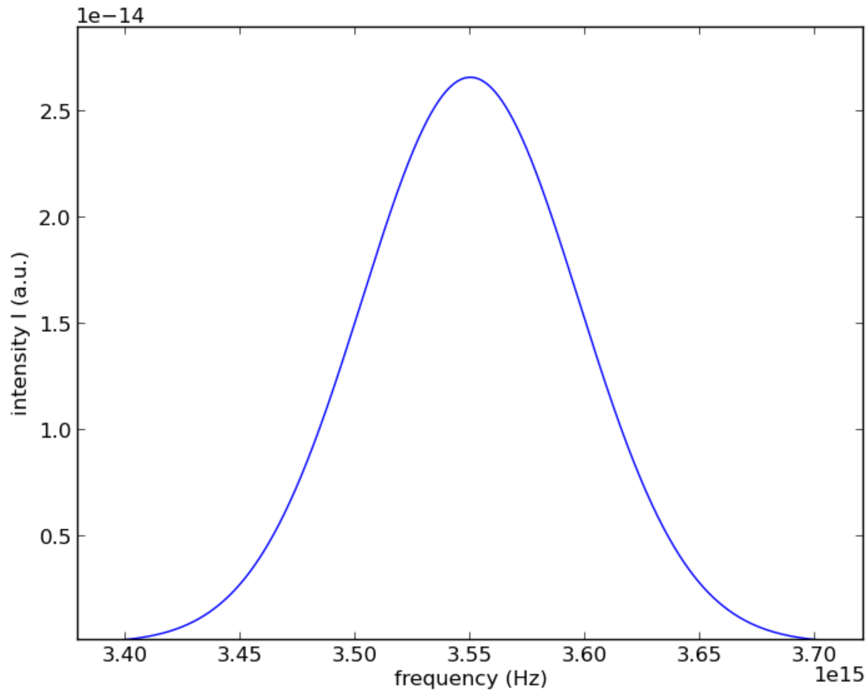

Figure 2.4: Simulated intensity of a Gaussian laser pulse plotted against frequency

The central frequency is 3.55 Hz and the full width at half maximum of the curve is 0.1 fs.

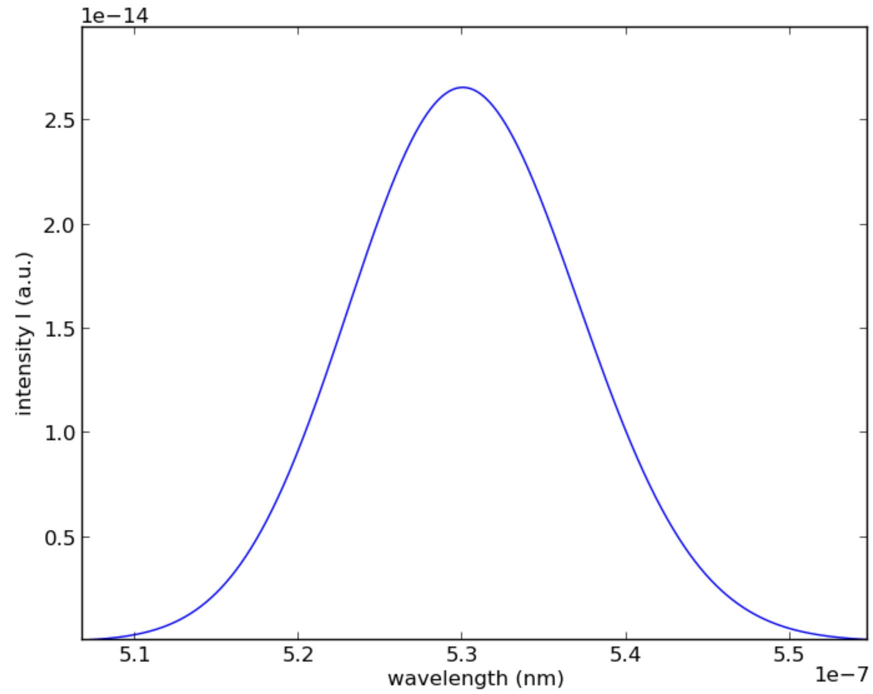

Figure 2.5: Simulated intensity of a Gaussian laser pulse plotted against its wavelength

2.3 Intensity Autocorrelation

The intensity autocorrelation, $A_{\tau}^{(2)}$ is the result of measuring a pulse against itself in the time domain. This is the process of splitting a pulse into two replicas of itself, variably delaying one of the replicas with respect to the other, and spatially overlapping the two pulses in a second harmonic generation (SHG) crystal. The second harmonic generation crystal will produce light at twice the frequency of the input light with a field envelope given by

$$E(t, \tau) \propto E(t)E(t - \tau). \tag{2.15}$$

The intensisty of this field is proportional to the product of the intensities of the two input pulses:

$$I(t, \tau) \propto I(t)I(t - \tau). \tag{2.16}$$

Since common detectors are too slow to resolve this beam in time, they will measure

$$A_{(\tau)}^{(2)} = \int_{-\infty}^{\infty} I(t)I(t - \tau)dt. \tag{2.17}$$

$A_{(\tau)}^{(2)}$ is second order autocorrelation, the subscript (2) indicate second order autocorrelation, third order generation is also possible.

3. Experiment

3.1 Experiment Set Up

The following set up is a schematic diagram of a Nonlinear Optical Parametric Amplifier (NOPA). A NOPA is an optical device or set up which receives an input signal and generate an output signal with higher optical power based on nonlinear interactions.

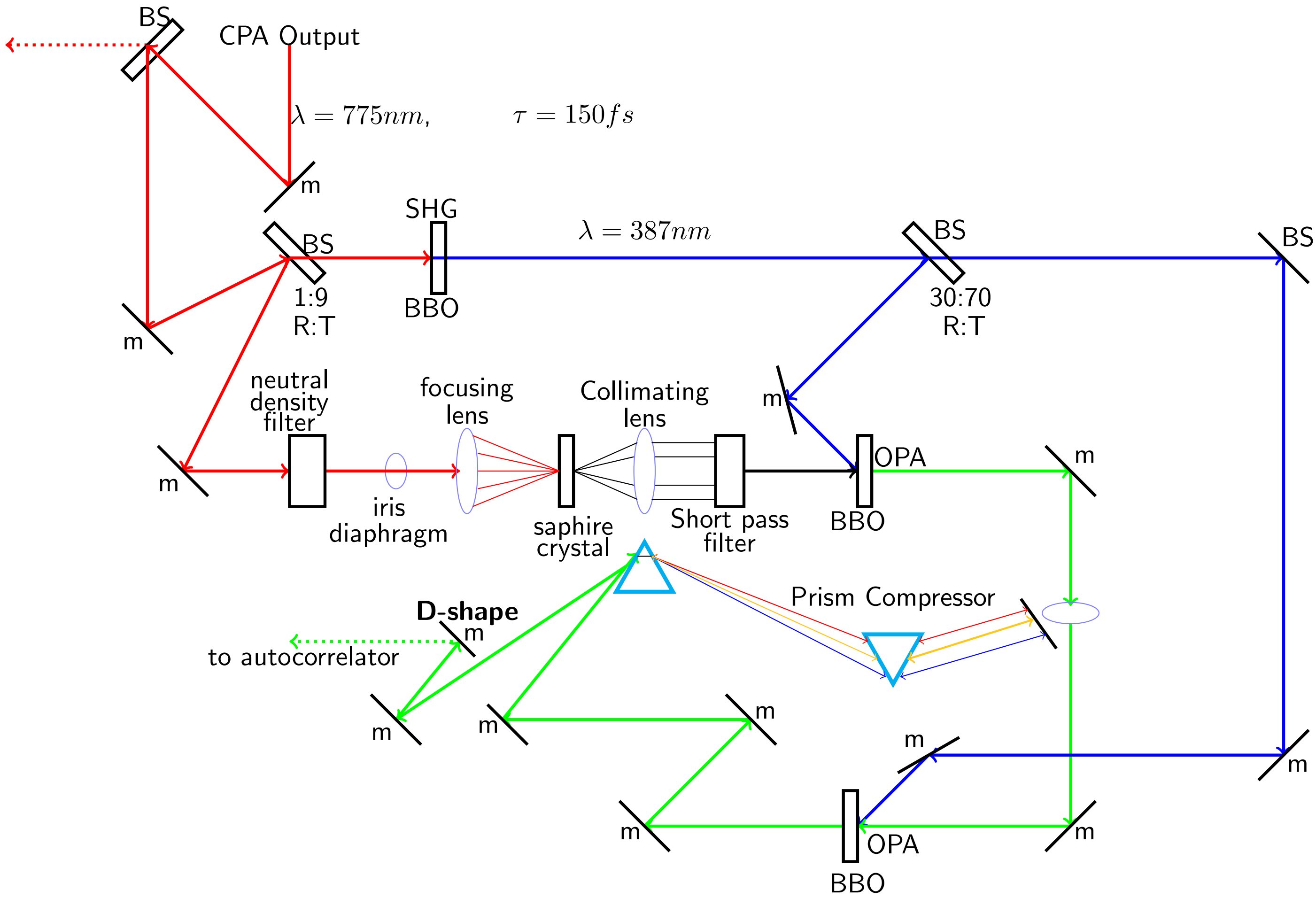

Figure 3.1: An Experimental set up of a Nonlinear Optical Parametric Amplifier (NOPA)

Input stage

In this stage a beam of a 775 nm wavelength laser, is emitted by Ti:sapphire laser (also known as $Ti : Al_2O_3$ laser, titanium sapphire laser, or simply Ti:sapphires) and is splitted by a beam splitter. The pulse is further splitted with approximately 4% of it been reflected. The frequency of our transmitted pulse is doubled in a Second Harmonic Generation (SHG), a $0.77mm$ Beta Barium Borate ($\beta - BaB_2O_4$) crystal and is used as our pump pulse.

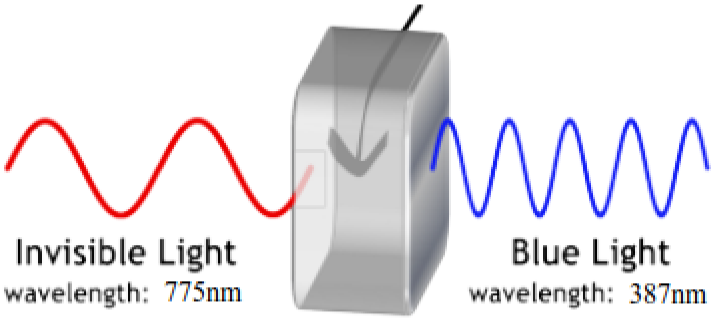

Figure 3.2: Frequency is doubled inside a nonlinear crystal Troiano (May 14, 2013)

The reflected pulse at this point, low in intensity, compare to our pump pulse at a ratio of $1:9$ is reflected by a broadband mirror into the white light generation stage.

3.2 White Light generation Stage

The term white light refers to a continuous spectrum, including all wavelengths from green to red and way into the infrared spectral range.

To generate white light we start by intensity control of the beam, that is the modulation of a 775 nm pulse by passing it through a neutral density filter. Upon exiting the filter the beam pass through an iris diaphram, which controls the numerical aperture of the incident beam. At this point it is important to note that the beam intensity and aperture are controlled in order to create white light filament with a discretized energy of approximately $2\mu J$. After generating white light, a 30 mm plano-convex fused silica lens is used to align and collimate the beam.Here again a filter is used to improve relative intensity distribution of the white generated light. The filter blocks all light above 700 nm of wavelength.

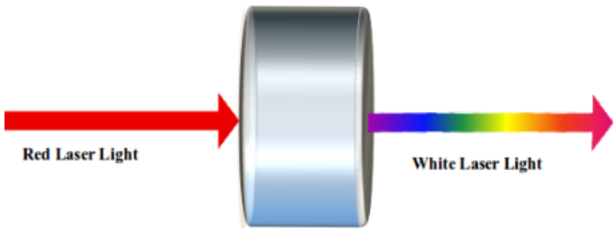

Figure 3.3: White generation inside a sapphire crystal (Troiano, May 14, 2013)

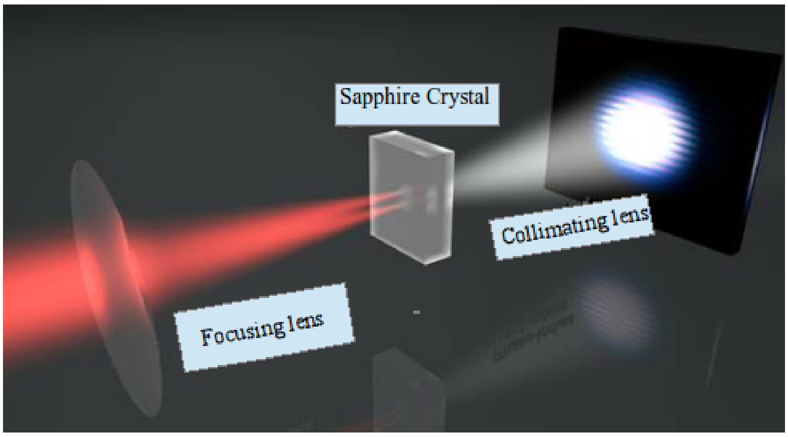

Figure 3.4: A focusing lens, sapphire crystal and, collimating lens in the white light generation stage(**?**)

3.3 First amplification stage of white light

22% of the frequency doubled blue pump pulse from the second harmonic generator (SHG) is reflected by a beam splitter on to a focusing mirror with focusing length of 150 mm. This highly reflective focusing mirror than reflects the blue laser light into a 1 mm BBO crystal where it is overlapped with the seed light from the white light stage. Diagram and explaination of white light amplification inside the optical parametric amplifier (OPA) is given in later section.

Again, it is important to note that in order to avoid damaging of the crystal, the pump light is focus in front of the crystal such that the diameter of the pump and the seed pulses match within the crystal.

3.4 Second Amplification Stage

In this stage the amplified seed pulse is directed past two silver mirrors and focus by a silica lens onto a BBO crystal. Coming from another path is the transmitted pump light which is reflected via two steering mirrors and a focusing mirror at an adjustable angle unto the BBO crystal. Here the pre-amplified white light from the first amplification stage and the pump light are carefully phase-matched to avoid getting an output power that is the sum of two individual powers from the amplification of different spectral regions. The output is therefore, a multiplicative amplification.

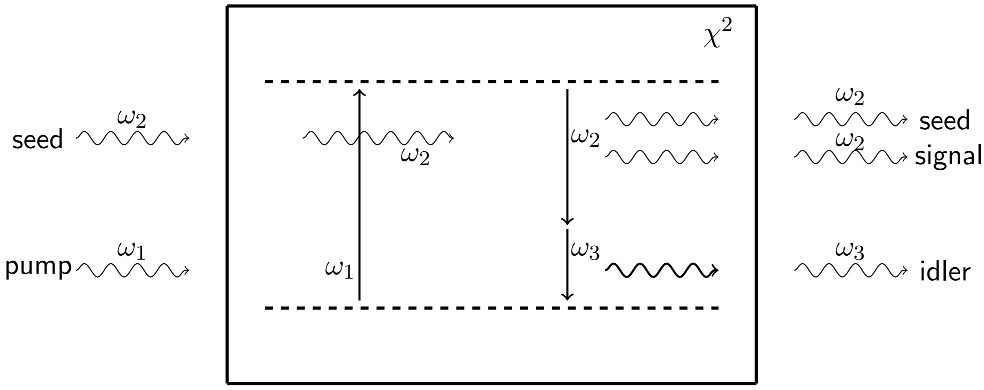

Figure 3.5: **Beta Barium Borate ($\beta - BaB_2O_4$) crystal**:

Process of white light amplification. In the BBO crystal a pump pulse of frequency ω_1 is excited by a weaker seed pulse of frequency ω_2. This stimulation results in the creation of a signal pulse of frequency ω_2 and an idler pulse of frequency ω_3 due to the anihilation of the pump pulse. That is, $\omega_1 = \omega_2 + \omega_3$.

3.5 The Output and Compressor

Different colors propagate at different velocities in a medium experiencing an effect called group-delay dispersion (GDD). Since ultrashort pulses have broad spectra, GDD results in significant pulse lengthening or intensity increase.

To compensate for the GDD of the NOPA output, a pulse compressor, a silica Brewster prism pair is used. A prism pulse compressor is an optical device used to shortened the duration of a positively chirped ultrashort laser pulse by giving different wavelength components different time delays. A positively chirped ultrashort laser pulse is a pulse in which light with longer wavelength travels faster than shorter wavelength light. Overall the prism compressor makes wavelength components of a laser pulse overlap with each other, thus causing a shorter pulse.

Prism 1 in Figure (3.1) refracts different wavelength components of light to slightly different angles causing these different wavelength components to travel in different paths but in parallel directions. Prism 2 also refracts all components as was done by prism 1 onto a reflecting mirror.These refracted wavelength components are retro reflected by the mirror, again to let them propagate in parallel directions retracing their paths.

3.6 Autocorrelator

The amplified pulse from the NOPA is directed to a BBO crystal via reflecting mirrors and a pair of irises. The irises align the beam to hit the two D-shape mirrors precisely at the middle of the mirrors. The two D-shape mirrors split the incoming NOPA output pulse into two identical halves and reflect the split pulses onto a focusing mirror which in turn focus the pulses into a BBO crystal. In the BBO crystal the pulses are convoluted and frequency is doubled. Emerging from the BBO crystal we observe three outcomes; pulse a interact with itself, pulse a interact with pulse b and, pulse b interacts with itself. The use of a and b here is only meant to indicate two pulses and don't refer to chronological sequence. The frequency doubled pulses are then filtered to block visible light and transmit UV light pulses and remaining light is then focus on to a detector by a plano convex focusing lens.

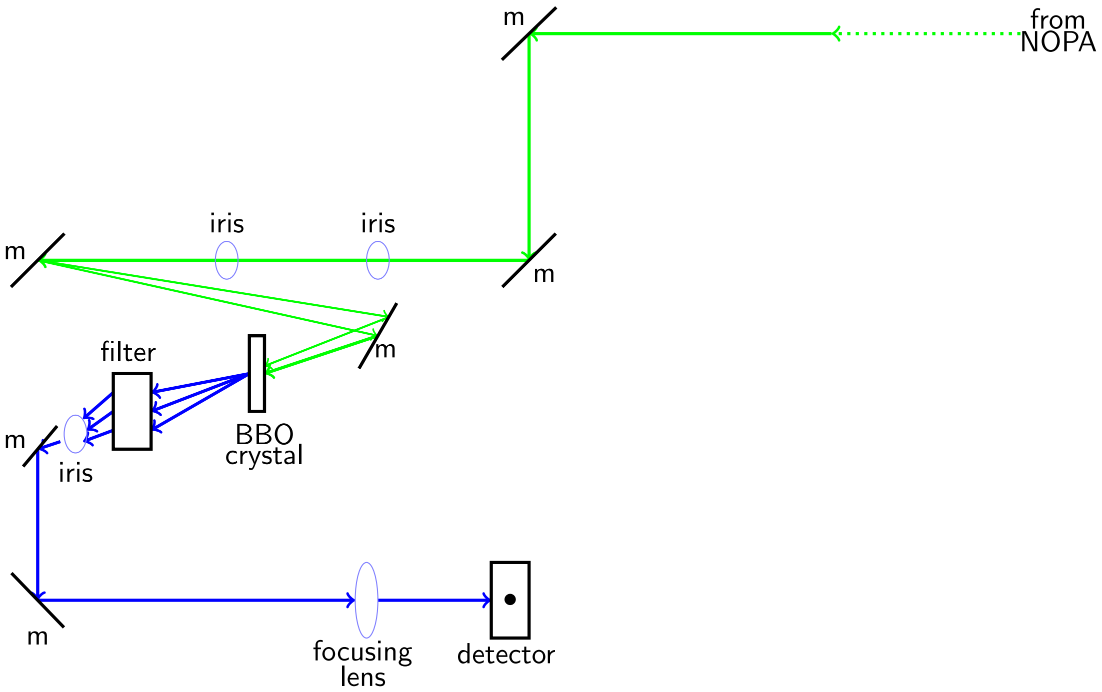

Figure 3.6: A Sketch of the autocorrelator. Depicted is the path of the NOPA output pulse to the autocorrelator. A photodiode detector register the signal from the autocorrelation.

4. Results and Measurements

We used an autocorrelator to measure the intensity of the convoluted pulse. We achieved this by splitting the frequency doubled pulse using an autocorrelator, time delaying one of the splitted pulses than recombining them in a SHG crystal. The out put signal current from the autocorrelator was than converted into voltage and read on a digital oscilloscope. This signal out put is plotted in the figure below against time.

4.1 Plot of Autocorrelation Signal.

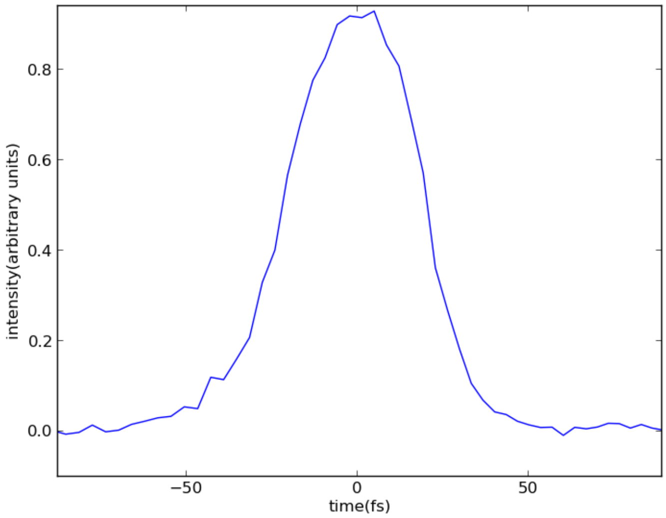

Figure 4.1: Second order Intensity (a.u.) autocorrelation signal of a 530 nm pulse plotted against time (fs).

This autocorrelation signal can be fitted with a Gaussian as shown in the following plot. The accuracy of this measurement is limited to about 25% due to several experimental parameters, which are not trivial to quantify.

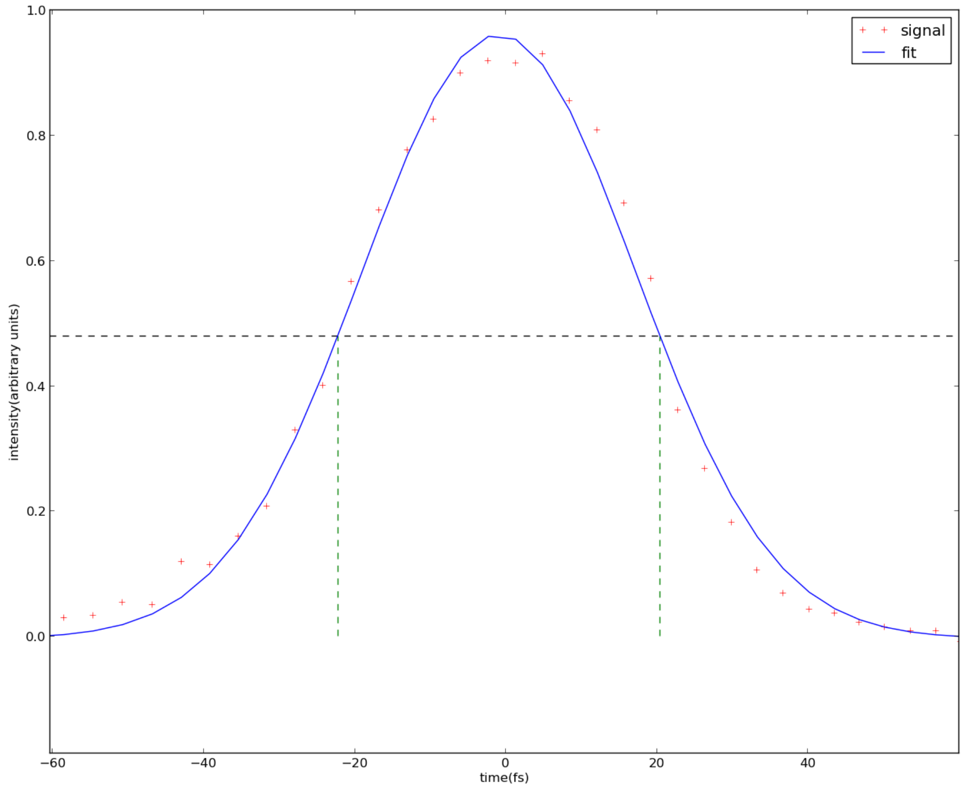

Figure 4.2: Intensity (a.u.) of a 530nm pulse of a Second order autocorrelation signal from the autocorrelator plotted against time (fs) and fitted against a Gaussain function. Horizontal dashed lines intersect vertical dashed lines at points where the intensity value drop to $1/e$.

From the plot we extract the amplitude and standard deviation.

`amplitude = 9.65e-1, standard deviation = 18`

4.2 Calculating the Pulse Duration.

From the plot of the pulse intensity against time in Figure (4.2) we calculate the full width at half maximum of the pulse (τ_{FWHM}),

Using equation (2.12) FWHM $= \tau = 2 \cdot \sigma \cdot \sqrt{2ln2}$ we have

$$\tau_{AC} = 2 \cdot \sigma \cdot \sqrt{2ln2},$$

$$\tau_{AC} = 2 \cdot 18.1 \cdot \sqrt{2ln2},$$

$$\tau_{AC} = 42.6 \text{ fs}$$

Where τ_{AC} is the width of the autocorrelation function.

Taking into account convolution we have

$$\tau_{AC} = \frac{1}{D_{AC}} \cdot \tau. \tag{4.1}$$

τ is the pulse width and D_{AC} is the deconvolution factor. For a Gaussian pulse D_{AC} is given by

$$D_{AC} = \frac{1}{\sqrt{2}}, \tag{4.2}$$

Therefore,

$$\tau = \frac{1}{\sqrt{2}} \cdot \tau_{AC},$$

$$\tau = \frac{1}{\sqrt{2}} \cdot 42.6 \text{ fs}.$$

Hence the pulse duration,

$$\tau = 30.1 \text{ fs}.$$

4.3 Spectrum

As an extra we calculate the bandwidth spectrum and also fit it with a Gaussian function.

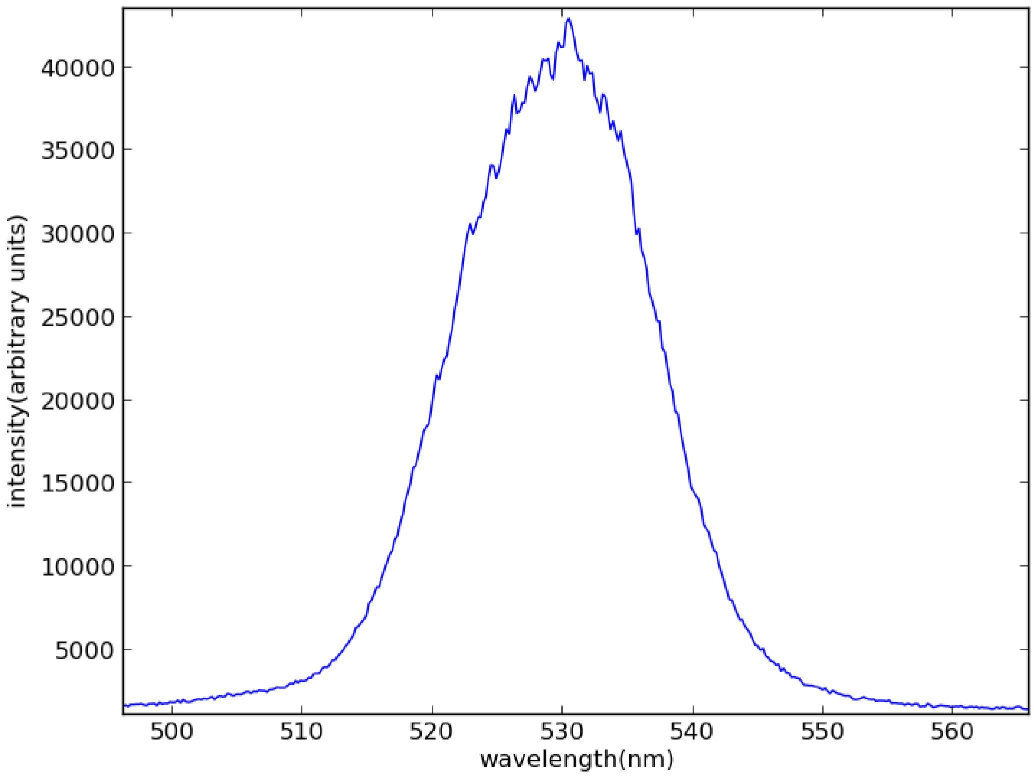

Figure 4.3: Spectrum of a < 40 fs, 530 nm pulse plotted as intensity (a.u) versus wavelength (nm).

The uncertainty of the experimentally determined spectral width is about 10% again due to several experimental parameters, of which probably the most influencial one is the flutuation of the pulse spectrum from pulse to pulse.

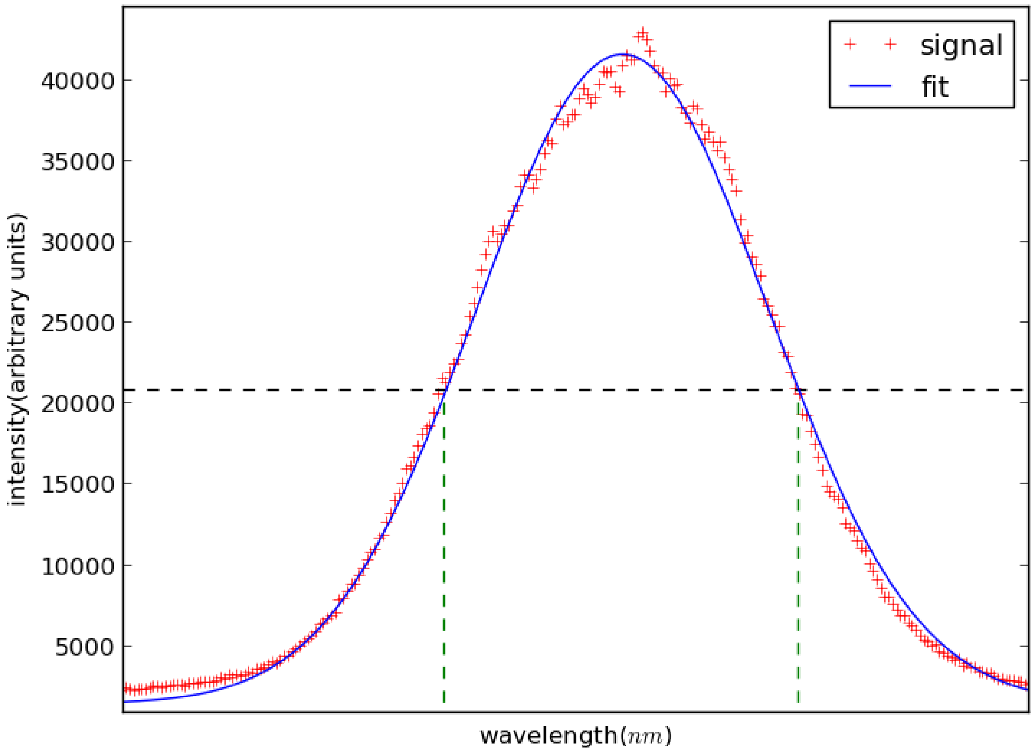

Figure 4.4: The Spectrum above in Figure 4.3 fitted against a Gaussian function. Horizontal dashed lines intersect vertical dashed lines at points where the intensity value drop to $1/e$.

From spectrum plot in Figure 4.4 we extract values of the amplitude of the signal, central wavelength of the pulse, and the standard deviation.

`amplitude = 4.02e4, central wavelength = 530e2, standard deviation = 7`

4.3.1 Calculating the Bandwidth.. Bandwidth is the width of some frequency or wavelength range. A common definition of bandwidth is the full width at half maximum (FWHM).

Spectral width in terms of wavelength of the ultrashort laser pulse plotted in Figure (4.4) is calculated below. Again we use equation 2.12 (FWHM $= \tau = 2 \cdot \sigma \cdot \sqrt{2ln2}$) to calculate the FWHM of the spectrum

$$\Delta\lambda = 2 \cdot \sigma \cdot \sqrt{2ln2},$$

$$\Delta\lambda = 2 \cdot (7) \cdot \sqrt{2ln2},$$

$$\Delta\lambda = 17.4 \text{ nm}.$$

The spectral width may also be specified in terms of frequency. By definition,

$$\omega = \frac{c}{\lambda}, \tag{4.3}$$

but for small wave length interval we have

$$\begin{aligned}\Delta\omega &= \frac{c}{\lambda^2} \cdot \Delta\lambda \\ &= \frac{2.99 \cdot 10^8\ m/s}{(5.29 \cdot 10^2\ \text{nm})^2} \cdot 17.4\ \text{nm} \\ &= 1.87 \cdot 10^{13}\ \text{Hz} \\ &= 18.7\ \text{THz}.\end{aligned} \tag{4.4}$$

5. Conclusion

Using data from the experiment, I plotted intensity against time and extracted the pulse duration of the ultrashort laser pulse as full width at half maximum of the autocorrelation trace. We determined the pulse duration to be 30.1 fs. We also plotted intensity against wavelength and calculated the wavelength spectral width of the pulse to be 17.4 nm and the frequency spectral width as $18.t$ THz.

Now, we check if our pulse is transform limited. Transform limit or Fourier limit is the lowest possible limit for a pulse duration for a given optical spectrum of a pulse. The minimum time bandwidth product as shown in equation 2.14 is 0.441 for a Gaussian pulse. This time bandwidth product indicates if a pulse is transform limited.

For my pulse the time bandwidth product is

$$\begin{aligned} \Delta\omega \cdot \Delta\tau &= 30.1 \text{ fs} \cdot 18.7 \text{ THz} \\ &= 0.562 \end{aligned}$$

This result means that our ultrashort laser pulse is nearly transform limited.

Our measured pulse is longer than the transform limited pulse due to chromatic dispersion. Frequency is dependent on phase velocity in transparent medium. A related quantitative measure of chromatic dispersion is the group velocity dispersion. Optical filters can also affect the spectral width or shape of an ultralshort pulse. Reduction of the spectral width may result into temporal broadening. Light reflection from mirrors also causes dispersion. This temporal broadening occurs because the reflected pulse has specific reflectivity at different wavelengths of the spectrum.

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