Higgs-Kibble Mechanism and the Standard Model

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Abstract

The Standard Model Theory is reviewed in a nutshell, and the problem of the massless gauge fields bosons and the idea of spontaneous symmetry breaking are stated.Then, the Higgs Mechanism is discussed and the masses of the gauge bosons of the electroweak fields are derived.Their theoretical values are obtained and compared with the experimental ones. Beside that, the masses of the fermions where the Yukawa coupling is present, are also derived. Moreover, the experiment used to find the Higgs boson, and the results are briefly reviewed.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Saga Elsayed Ibrahim Abdalla, 22 May 2014

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1. Introduction

As we get deeper inside matter, we pass through various structures formed by forces that holds them together. Take the atom as an example, where the attraction force between the electron e^- and the proton is what holds it together. In order to describe this force, quantum theory could be employed or more precisely, the Standard Model where the attraction is caused by virtual particles[1]. These particles perfectly fill the space forming a field[2]. Every single virtual particle is a force mediator of that field, and the strength at any point is the density of the field particle at that point, which is given by the Lagrangian density.

The Standard Model combines Quantum electrodynamics (which describes the electromagnetic interactions e.g. atoms interaction) with Quantum Chromodynamics (which describes the strong interactions e.g. quark and anti-quark attraction) and the weak force theory (which describes the weak interactions e.g. β-decay).

The Standard Model has three main classifications leptons, quarks and bosons. The leptons are the spin-$\frac{1}{2}$ fermions and their massless neutrinos. There are three types of generations, the electrons, muons and tauon including their neutrinos. Due to the fact that they occur in pairs, they are written in doublets, i.e.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}.$$

All these form the non-strong interacting particles. On the other hand, there exist six quarks which occur in pairs as well, written as

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix},$$

where u and d are the up and down quarks, c and s are the respective charm and strange quarks, and t and b are the top and bottom quarks, respectively. Quarks combine to make the *hadrons*, which is a family consisting of two members; the *baryons*, which are a combination of three quarks (qqq), such as neutrons, and the *mesons* which are the combination of a quark and an anti-quark $(q\bar{q})$, such as pions.

Finally, there are bosons such as photon (γ), which is the gauge boson (mediator) of the electromagnetic force, gluon (g), the gauge boson of the strong force and $W^\pm$ and Z bosons which are the gauge bosons of the weak force. Each of these mediators has its own range of force which is inversely proportional to its mass. Since photons are massless they do not have a finite range of propagation unlike the massive $W^\pm$ and Z which have a very short range. The gravitational force has a negligible contribution to small scale interactions, it has been ignored (Martin and Shaw, 1992; Griffiths, 2008).

To describe the interaction between the fundamental fields, they have to obey the gauge principles where we can use Noether's theorem which states that "For Every invariant continuous transformation of the field, there is a combination of the function and its derivative which is constant as the time changes (conserved)". "The importance of this theorem is that it relates conservation laws to symmetries in the Lagrangian of the field".(Gross, 2004; McMahon, 2008).

To describe an electron in an electromagnetic field (complex field), the Lagrangian corresponding to the Dirac equation

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi, \tag{1.0.1}$$

[1] They call virtual particles because they violate the conservation of energy for a very short time in order to exist by the energy fluctuation allowed by Heisenberg uncertainty principle $\Delta E\Delta t \geq \hbar$ and they gone because of the same principle.

[2] A field is used to describe something that have different values as we move in it space

where ψ and $\bar{\psi}$ are the field and the complex conjugate of the field, respectively, and m is the mass of the field. The invariant transformation of ψ can be written as

$$\psi \to e^{i\alpha}\psi, \tag{1.0.2}$$

where $e^{i\alpha}$ is the phase transformation and α is a real constant. Observe that equation (1.0.2) is of the form

$$\phi(x) \to U(\alpha)\phi(x) \equiv e^{i\alpha}\phi(x), \qquad \alpha \in \mathbb{R},$$

where x is the space component of the space-time vector $x^\mu = (x, t)$.

Which is in the form of the unitary abelian group $U(1)$. The transformation described in equation (1.0.2) does not give any physical meaning of α since it is constant in all the transformations. Such a transformation is termed a *global transformation* and thus $U(1)$ is globally conserved, but, if a physics is conserved only locally it transform as

$$\psi \to e^{i\alpha(x)}\psi, \tag{1.0.3}$$

where $\alpha(x)$ is a function of space and time which varies as we move in the field space. Plugging this transformation in equation (1.0.1) gives

$$\begin{aligned}
\tilde{\mathcal{L}} &= ie^{-i\alpha(x)}\bar{\psi}\gamma_\mu\left[ie^{i\alpha(x)}\psi\partial^\mu\alpha + e^{i\alpha(x)}\partial^\mu\psi\right] - me^{-i\alpha(x)}\bar{\psi}e^{i\alpha(x)}\psi,\\
&= -\bar{\psi}\gamma_\mu\psi\partial^\mu\alpha + i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi,\\
&= -\bar{\psi}\gamma_\mu\psi\partial^\mu\alpha + \mathcal{L}.
\end{aligned} \tag{1.0.4}$$

Observe the extra term breaks the invariance of the field.This can be restored by introducing the covariant derivative[3] expressed in terms of a 4-vector field A_μ which is chosen to transform in a way that cancels out the extra term in equation (1.0.4)

$$\begin{aligned}
D_\mu &\equiv \partial_\mu - ieA_\mu,\\
A_\mu &\to A_\mu + \frac{1}{e}\partial_\mu\alpha.
\end{aligned} \tag{1.0.5}$$

Therefore, equation (1.0.1) can be rewritten as

$$\begin{aligned}
\mathcal{L} &= i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi,\\
&= i\bar{\psi}\gamma_\mu\left[\partial^\mu - ieA^\mu\right]\psi - m\bar{\psi}\psi,\\
&= i\bar{\psi}\gamma_\mu\partial^\mu\psi + e\gamma_\mu\bar{\psi}\psi A^\mu - m\bar{\psi}\psi.
\end{aligned} \tag{1.0.6}$$

Thus, the transformation of the Dirac equation Lagrangian is (Halzen and Martin, 1984)

$$\begin{aligned}
\tilde{\mathcal{L}} &= ie^{-i\alpha(x)}\bar{\psi}\gamma_\mu\partial^\mu\left(e^{i\alpha(x)}\psi\right) + e\gamma_\mu e^{-i\alpha(x)}\bar{\psi}e^{i\alpha(x)}\psi A^\mu - me^{-i\alpha(x)}\bar{\psi}e^{i\alpha(x)}\psi,\\
&= i\bar{\psi}\gamma_\mu\partial^\mu\psi + e\gamma_\mu\bar{\psi}\psi A^\mu - m\bar{\psi}\psi = \mathcal{L}.
\end{aligned} \tag{1.0.7}$$

[3] Note that definition of the contravariant derivative is the same of the covariant because the metric tensor cancel out and it becomes like we are only swapping the indices.

The Mass Term in the Lagrangian

It is a measurable quantity in the Lagrangian field equation and invariant under the gauge transformation. It found out that it is the quadratic term in the field Lagrangian multiplied by a square of some constant. Take the Proca Lagrangian as an example

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\kappa^2 A_\mu A^\mu, \\ =& -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}\kappa^2 A_\mu A^\mu,\end{aligned} \tag{1.0.8}$$

where $F^{\mu\nu}$ is the strength tensor.

Then, the Lagrangian field equation is (see appendix (A.1))

$$\frac{\partial \mathcal{L}}{\partial A_\mu} - \frac{\partial}{\partial x^\nu}\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \Longrightarrow -\partial_\nu F^{\mu\nu} - \frac{1}{2}\kappa^2 A^\mu = 0. \tag{1.0.9}$$

From equation (1.0.9),

$$\partial_\nu \partial^\nu A^\mu - \frac{1}{2}\kappa^2 A^\mu = 0 \Longrightarrow \partial_\nu \partial^\nu A^\mu = \frac{1}{2}\kappa^2 A^\mu, \tag{1.0.10}$$

Alternatively, in terms of the 4-momentum tensor P^μ[4], the above equation can be written as

$$P_\mu P^\mu A^\mu = -m^2 A^\mu = \frac{1}{2}\kappa^2 A^\mu, \tag{1.0.11}$$

Therefore, the mass term in (1.0.8) is

$$-\frac{1}{2}\kappa^2 A_\mu A^\mu$$

It is not difficult to check that this strength tensor is invariant. Using the gauge field transformation (1.0.5) yields

$$\begin{aligned}\tilde{F_{\mu\nu}} =& \partial_\mu A_\nu + \frac{1}{e}\partial_\mu\partial_\nu\alpha - \left[\partial_\nu A_\mu + \frac{1}{e}\partial_\nu\partial_\mu\alpha\right], \\ =& \partial_\mu A_\nu + \frac{1}{e}\partial_\mu\partial_\nu\alpha - \partial_\nu A_\mu - \frac{1}{e}\partial_\nu\partial_\mu\alpha = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}.\end{aligned}$$

Finally, the *locally invariant* Lagrangian of the $U(1)$ transformation is

$$\begin{aligned}\mathcal{L} =& i\bar{\psi}\gamma_\mu\partial^\mu\psi + e\gamma_\mu\bar{\psi}\psi A^\mu - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ =& \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi + e\bar{\psi}\gamma_\mu A^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.\end{aligned} \tag{1.0.12}$$

The above Lagrangian has no vector field mass term $\frac{1}{2}m^2 A_\mu A^\mu$, where it addition break the invariance of the Lagrangian;

$$\frac{1}{2}m^2 A_\mu A^\mu = \frac{1}{2}m^2\left[A_\mu + \frac{1}{e}\partial_\mu\alpha\right]\left[A^\mu + \frac{1}{e}\partial^\mu\alpha\right] \neq \frac{1}{2}m^2 A_\mu A^\mu.$$

Hence, equation (1.0.12) described formalism, that is successful for Quantum Electrodynamics (QED), but cannot applied straight forward to the weak interactions, where the the massive gauge bosons $W^\pm$ and Z are massless. We will see in the next chapter how this problem is solved by inventing a new idea called the spontaneous symmetry breaking of the local gauge symmetry, this idea led to the so-called *Higgs mechanism* which explains the origin of mass of elementary particles (Halzen and Martin, 1984; van Vulpen, 2013; Griffiths, 2008).

[4]Note that $P^\mu = (-E, \boldsymbol{p})$, and $E^2 - \boldsymbol{p}^2 = m^2$, where E is the energy and $\boldsymbol{p}$ is the momentum vector.

2. Spontaneous Symmetry Breaking and the Need of New Mechanism

2.1 Spontaneous Symmetry Breaking

Imagine a smooth golf ball placed on its stand, this system is said to be in ground state and symmetric because the view is the same for an observer setting on the top of the ball (grass is everywhere around). Now if we perturbed the ball it will immediately move to real ground states (anywhere in the grass around the stand) but by choosing any point of the ground states the system becomes non-symmetric and undergoes spontaneously symmetry breaking.

Translating that to our case, we say that some Lagrangian that describe a field have unstable ground states which keep it invariant, while it has stable ground states and by choosing one of those stable points the system is spontaneous breaking the symmetry. Take the (ϕ^4) Lagrangian to describe a scalar self-interacting field with internal degrees of freedom

$$\mathcal{L} = T - V = (\partial_\mu \phi)^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2, \tag{2.1.1}$$

which is invariant under the transformation $\phi(x) \to e^{i\alpha_a \tau_a}\phi(x)$, where the fields are given by the doublet

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{2.1.2}$$

and then our final Lagrangian is

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\left[\partial_\mu \phi_1^\dagger \partial^\mu \phi_1 + \partial_\mu \phi_2^\dagger \partial^\mu \phi_2 + \partial_\mu \phi_3^\dagger \partial^\mu \phi_3 + \partial_\mu \phi_4^\dagger \partial^\mu \phi_4\right] - \frac{1}{2}\mu^2\left[\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right] \\ & - \frac{1}{4}\lambda\left[\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right]^2. \end{aligned} \tag{2.1.3}$$

The ground state of the Lagrangian is given by

$$\begin{aligned} &\frac{\partial^4 V(\phi_1, \phi_2, \phi_3, \phi_4)}{\partial\phi_1 \partial\phi_2 \partial\phi_3 \partial\phi_4} = 0, \\ &\frac{\partial^4}{\partial\phi_1 \partial\phi_2 \partial\phi_3 \partial\phi_4}\left(\frac{1}{2}\mu^2\left[\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right] + \frac{1}{4}\lambda\left[\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right]^2\right) = 0, \\ &\mu^2(\phi_1 + \phi_2 + \phi_3 + \phi_4) + \lambda(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)(\phi_1 + \phi_2 + \phi_3 + \phi_4) = 0. \end{aligned} \tag{2.1.4}$$

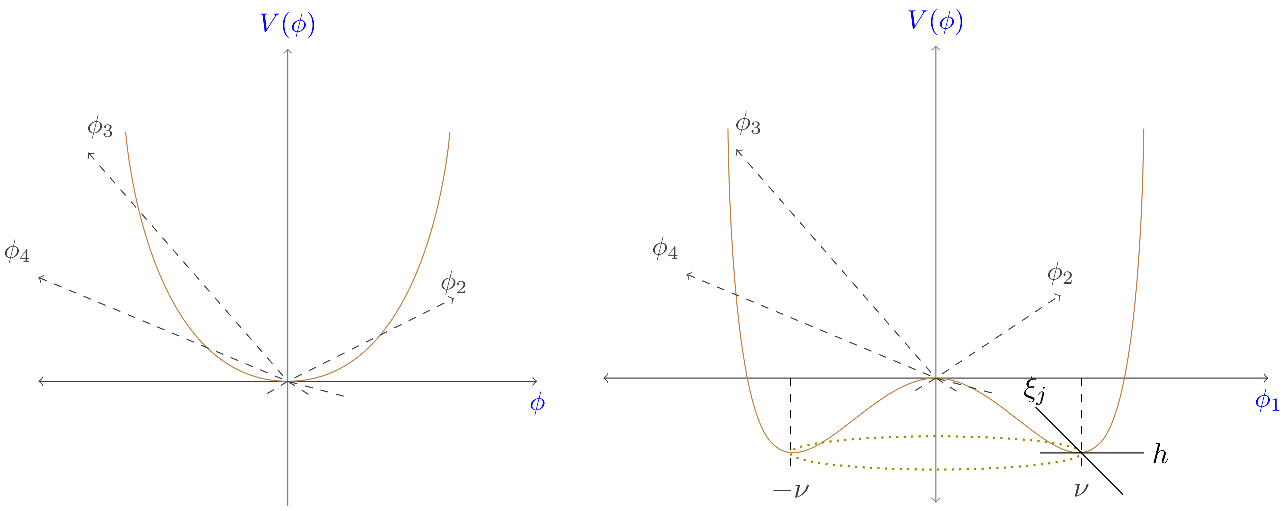
and so the ground states of the Lagrangian (2.1.5) are

$$(\phi_1 + \phi_2 + \phi_3 + \phi_4) = \sum_{i=1}^{4} \phi_i = 0, \tag{2.1.5}$$

or

$$(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \sum_{i=1}^{4} \phi_i^2 = \frac{-\mu^2}{\lambda} = v^2, \tag{2.1.6}$$

The first case corresponds to $\mu^2 > 0$, the second corresponds to $\mu^2 < 0$ and $\lambda > 0$ but we cannot get any physical description of the system by using them and so we perturbate around the them and study the system. It is clear that it is useless to perturb around (2.1.5) and so we will only focus on the second case which is a 4-dimensional sphere with radius v^2 (2.1.6). A 2-dimensional plot of the two ground states Figure (2.1a) and Figure (2.1b) shows that the first ground state is symmetric but, the second plot shows stable points $\left(\sqrt{\sum_{i=1}^{4}\phi_i^2} = v \text{ and } \sqrt{\sum_{i=1}^{4}\phi_i^2} = -v\right)$ and by choosing one of them the system spontaneously breaks the symmetry.



(a) The $\mu^2 > 0$ case.

(b) The $\mu^2 < 0$ and $\lambda > 0$ case.

Figure 2.1: The ground states of the system (Halzen and Martin, 1984).

Now if we chose $\sqrt{\sum_{i=1}^{4}\phi_i^2} = v$ to investigate the particle, it is convenient to expand around v by defining new real fields $\xi_i(x)$ where $i = 1, 2, 3, 4$, choose $\xi_3(x) = (v + h(x))$, where $h(x)$ is the Higgs field, (2.1.2) becomes

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_1 + i\xi_2 \\ (v+h) + i\xi_4 \end{pmatrix}. \tag{2.1.7}$$

Then, we can rewrite the Lagrangian[1] in (2.1.1) in terms of our new field as

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\left[\partial_\mu \xi_1^\dagger \partial^\mu \xi_1 + \partial_\mu \xi_2^\dagger \partial^\mu \xi_2 + \partial_\mu (v+h)^\dagger \partial^\mu (v+h) + \partial_\mu \xi_4^\dagger \partial^\mu \xi_4\right] \\ & - \frac{1}{2}\mu^2\left[\xi_1^2 + \xi_2^2 + (v+h)^2 + \xi_4^2\right] - \frac{1}{4}\lambda\left[\xi_1^2 + \xi_2^2 + (v+h)^2 + \xi_4^2\right]^2. \end{aligned} \tag{2.1.8}$$

Then the final Lagrangian is[2] (see appendix (A.2))

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\partial_\mu \xi_1 \partial^\mu \xi_1 + \frac{1}{2}\partial_\mu \xi_2 \partial^\mu \xi_2 + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \xi_4 \partial^\mu \xi_4 + \mu^2 h^2 - \mu^2 v h - \lambda v h^3 - \lambda v \xi_1^2 h - \lambda v \xi_2^2 h \\ & - \lambda v^3 h - \lambda v h \xi_4^2 - \frac{1}{2}\lambda h^2 \xi_1^2 - \frac{1}{2}\lambda \xi_2^2 h^2 - \frac{1}{2}\lambda \xi_4^2 h^2 - \frac{1}{4}\lambda \xi_1^4 - \frac{1}{4}\lambda \xi_2^4 - \frac{1}{4}\lambda h^4 - \frac{1}{4}\lambda \xi_4^4 \\ & - \frac{1}{2}\lambda \xi_1^2 \xi_2^2 - \frac{1}{2}\lambda \xi_1^2 \xi_4^2 - \frac{1}{2}\lambda \xi_2^2 \xi_4^2. \end{aligned} \tag{2.1.9}$$

[1] For simplicity we will write h instant of $h(x)$ and ξ instant of $\xi(x)$.
[2] After dropping the constant terms.

Equation (2.1.9) is the Lagrangian that described a particle in its ground state, with kinetic energy $\frac{1}{2}\partial_\mu \xi_i \partial^\mu \xi_i$, and self-interaction terms $\frac{1}{4}\lambda \xi_i^4$, $\lambda v h \xi_j^2$ where $j = 1, 2, 4$ and $\lambda v h^3$ represented respectively by four and three legs in Feynman diagrams Figure (2.2) (McMahon, 2008), Also, field interacting terms $\frac{1}{2}\lambda \xi_j^2 \xi_k^2$ where $k = 1, 2, 4$ and $j \neq k$. Finally, a mass term $\mu^2 h^2$, comparing it with the mass term in the Proca Lagrangian (1.0.8) observe that the mass of the Higgs field h is $m_h = \sqrt{-2}\mu$ and it can be seen that the fields ξ_j are massless. Now instead of having the masses of the three massive boson we have a three new massless bosons known as the Glodstone bosons.(Halzen and Martin, 1984)

(a) The three legs diagram $\lambda v h^3$ and $\lambda v h \xi_j$ (b) The four legs diagram $\frac{1}{4}\lambda \xi_i^4$.

Figure 2.2: The Feynman diagram of the self-interacting term(McMahon, 2008).

2.2 The Higgs Mechanism

Theoretical physicists had to find another mathematical way to formulate the equations to describe the actual masses of the elementary particles. A trick is needed to give masses to the assigned massive elementary particles and remain the massless and most important maintain the Lagrangian invariance.

That was achieved by the Scottish theoretical physicist Peter Higgs in 1964, who firstly, introduced a new real scalar field[3]with non-zero ground state named the Higgs field with a mediators called Higgs bosons. Secondly, he defined the Higgs field as the fluctuation about its non-trivial ground state (vacuum).

Finally, he added it to the field's Lagrangian that describe the interaction and chose the fields to be massless when the Higgs vacuum is trivial, by doing this trick the mass is generated to the assigned elementary particle while keeping the massless and maintaining the invariance of the Lagrangian.Those procedures are called the *Higgs Mechanism*.

2.3 Using the Higgs Mechanism to the Local Symmetry of SU(2)

The field (2.1.2) is transformed locally as

$$\phi(x) \rightarrow e^{i\alpha_a(x)\tau_a/2}\phi(x) \qquad a = 1, 2, 3, \tag{2.3.1}$$

[3]And so invariant under the any transformation.

where τ_a are the Pauli matrices with the following properties

$$\tau_a = \tau_a^\dagger = \vec{\boldsymbol{\tau}} \qquad \text{where } a = 1, 2, 3, \tag{2.3.2}$$
$$\vec{\boldsymbol{\tau}}^2 = \mathbb{I}, \qquad \text{where } \mathbb{I} \text{ is } 2\times 2 \text{ identity matrix,} \tag{2.3.3}$$
$$\tau_a\tau_b\tau_a = -\tau_b^*, \qquad \text{where } a, b = 1, 2, 3, \text{ and } a \neq b. \tag{2.3.4}$$

and the transformed Lagrangian (2.1.1) is

$$\begin{aligned}\tilde{\mathcal{L}} &= (\partial_\mu e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi)^\dagger(\partial^\mu e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi) - \mu^2\left[e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi\right]^\dagger e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi - \lambda\left(\left[e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi\right]^\dagger e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi\right)^2, \\ &= \left[\frac{i\vec{\boldsymbol{\tau}}}{2}\partial_\mu\alpha(x)e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi + e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\partial_\mu\phi\right]^\dagger\left[\frac{i\vec{\boldsymbol{\tau}}}{2}\partial^\mu\alpha(x)e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\phi + e^{i\alpha(x)\vec{\boldsymbol{\tau}}/2}\partial^\mu\phi\right] \\ &- \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \neq \mathcal{L}.\end{aligned} \tag{2.3.5}$$

The Lagrangian is not invariant under the transformation (2.3.1) and in order to restore this property we can write (2.3.1) as

$$\phi(x) \to U\phi(x), \tag{2.3.6}$$

where U is the unitary operator such that

$$UU^\dagger = \mathbb{I}, \tag{2.3.7}$$
$$\partial_\mu(UU^\dagger) = U^\dagger(\partial_\mu U) + U(\partial_\mu U^\dagger) = 0. \tag{2.3.8}$$

Define a new covariant derivative D_μ such that

$$D_\mu = \partial_\mu + i\frac{g}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu, \tag{2.3.9}$$

where $\vec{\mathbf{W}}_\mu$ are the three gauge fields defined as

$$\vec{\mathbf{W}}_\mu = (W_\mu^i), \quad i = 1, 2, 3, \tag{2.3.10}$$

and transformed as

$$\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu \to U(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu + \frac{i}{g}\partial_\mu)U^\dagger, \tag{2.3.11}$$

Then, the invariant transformed Lagrangian is

$$\begin{aligned}\tilde{\mathcal{L}} &= [UD_\mu\phi]^\dagger[UD^\mu\phi] - \mu^2[U\phi]^\dagger[U\phi] - \lambda\left[(U\phi)^\dagger(U\phi)\right]^2, \\ &= [D_\mu\phi]^\dagger U^\dagger U[D^\mu\phi] - \mu^2\phi^\dagger U^\dagger U\phi - \lambda[\phi^\dagger U^\dagger U\phi]^2 = [D_\mu\phi]^\dagger[D^\mu\phi] - \mu^2\phi^\dagger\phi - \lambda[\phi^\dagger\phi]^2 = \mathcal{L},\end{aligned} \tag{2.3.12}$$

and the final invariant Lagrangian for the Higgs field is

$$\mathcal{L} = \left[\partial_\mu\phi + i\frac{g}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\phi\right]^\dagger\left[\partial^\mu\phi + i\frac{g}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}^\mu\phi\right] - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{2.3.13}$$

There is need to add a kinetic energy term correspond to the gauge fields to say that the Lagrangian is describe a weak interaction. The kinetic energy have to be invariant under (2.3.11), it can be written as

$$\vec{\mathbf{W}}_{\mu\nu} = \partial_\mu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu) - \partial_\nu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu) - ig[\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu, \vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu], \tag{2.3.14}$$

where the commutator term appears because the $SU(2)$ is not abelian.

Using (2.3.7) and (2.3.8) and expanding the commutators we will find that the field tensor transform invariantly (see appendix (A.3))

$$\tilde{\vec{\mathbf{W}}}_{\mu\nu} = U\left(\partial_\mu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu) - \partial_\nu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu) - ig[\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu, \vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu]\right)U^\dagger = U\vec{\mathbf{W}}_{\mu\nu}U^\dagger. \tag{2.3.15}$$

This transform invariantly in the Lagrangian. (see appendix (A.3))

Finally, the invariant Lagrangian for the weak interaction is

$$\mathcal{L} = \left[\partial_\mu\phi + i\frac{g}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\phi\right]^\dagger\left[\partial^\mu\phi + i\frac{g}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}^\mu\phi\right] - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}\vec{\mathbf{W}}_{\mu\nu}.\vec{\mathbf{W}}^{\mu\nu}. \tag{2.3.16}$$

By using the Higgs mechanism for the case when $\mu^2 < 0$ and $\lambda > 0$ and choosing $\phi_1 = \phi_2 = \phi_4 = 0$ in (2.1.2) and expanding around a vacuum ψ_0, defined as

$$\psi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \tag{2.3.17}$$

Thus, (2.1.7) is rewritten in the ground state as

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix}. \tag{2.3.18}$$

Recall that expanding the field ϕ around the vacuum (2.1.5) does not solve problem of the massless Goldstone boson. "So the fluctuation of the vacuum in term of the three real gauge fields $\boldsymbol{\theta} = (\theta_1(x), \theta_2(x), \theta_3(x))$ and the Higgs field $h(x)$ is parametrize as"

$$\begin{aligned}\phi(x) &= \frac{1}{\sqrt{2}}e^{i\vec{\boldsymbol{\tau}}.\boldsymbol{\theta}(\boldsymbol{x})/v}\begin{pmatrix}0\\ v + h(x)\end{pmatrix} \approx \frac{1}{\sqrt{2}}(1 + i\vec{\boldsymbol{\tau}}.\boldsymbol{\theta}(\boldsymbol{x})/v)\begin{pmatrix}0\\ v + h(x)\end{pmatrix}\\ &\approx \frac{1}{\sqrt{2}}\begin{pmatrix}\theta_2(x) + i\theta_1(x)\\ v + h(x) - i\theta_3(x)\end{pmatrix} + \textit{ quadratic terms}.\end{aligned} \tag{2.3.19}$$

Observe that "the fields are completely parametrized by the deviation around the vacuum ψ_0 even in small perturbations" and then the Lagrangian is invariant[4]. Therefore, we can set the $\theta(x)$s' fields to be zero (with massless bosons) and we return to (2.3.18). Now substituting (2.3.18) into (2.3.16) we get

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu h(x) + ig\frac{1}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu(v + h(x)))^\dagger(\partial^\mu h(x) + ig\frac{1}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}^\mu(v + h(x))) - \frac{1}{2}\mu^2h^2 - \frac{1}{4}\lambda h^4 - \frac{1}{4}\vec{\mathbf{W}}_{\mu\nu}\vec{\mathbf{W}}^{\mu\nu},\\ &= \frac{1}{2}\partial_\mu h(x)\partial^\mu h(x) - \frac{g^2}{8}(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu)(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}^\mu) + \frac{g^2}{2}\left(\frac{1}{4}(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu)(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}^\mu) - \mu^2\right)h^2 - \frac{1}{4}\lambda h^4 - \frac{1}{4}\vec{\mathbf{W}}_{\mu\nu}\vec{\mathbf{W}}^{\mu\nu}.\end{aligned} \tag{2.3.20}$$

Notice that the gauge bosons are massive now and their masses are given by:

$$g^2v^2\frac{1}{8}\vec{\mathbf{W}}_\mu\vec{\mathbf{W}}^\mu = \frac{1}{8}g^2v^2\left[(W_\mu^1W_1^\mu) + (W_\mu^2W_2^\mu) + (W_\mu^3W_3^\mu)\right], \tag{2.3.21}$$

[4]Because the real fields θ_1, θ_2, and θ_3 will just represent a real shift in the invariant field.

compare this result by the gauge boson mass term $\frac{1}{2}M^2\vec{\mathbf{W}}_\mu\vec{\mathbf{W}}^\mu$ the mass of the boson is

$$\frac{1}{2}M_W^2 = \frac{1}{8}g^2v^2 \Longrightarrow M_W = \frac{1}{2}gv. \tag{2.3.22}$$

Recall that by using (2.1.7), only the massless Goldstone bosons with the two degrees of polarization[5](freedom) are generated, and by using the Higgs mechanism the gauge fields became massive or colloquially speaking, they have "eaten up" the Goldstone massless bosons and therefore gained a third degree of polarization which is the longitudinal polarization. That is why the massive fields are also called the vector fields.

Note that the number of degrees of freedom is conserved before and after the spontaneous symmetry breaking (the two cases in (2.1.5) and (2.1.6)). Counting them in the first case when $\mu^2 > 0$, there are four degrees of freedom for the field ϕ in order to have the value 0, and from (2.1.2) we see that the field ϕ is expressed as a doublet of two scalar field and two complex fields, which are massless in this case, and so we have eight degrees of freedom. Then, we have twelve degrees of freedom as total.

While in the second case when $\mu^2 < 0$ there are three massive gauge fields with three degrees of freedom for each and therefore nine degrees of freedom and two degrees of freedom from the massless photon field (see section (3.1)), finally, a single degree of freedom from the Higgs field h what sum up to twelve.

In conclusion, the spontaneous symmetry breaking has just rearranged those degrees in order to give mass to elementary particles. Now we will move to the electroweak model and we will see how to generate the approximate masses for weak bosons beside that the masses of the fermions (Halzen and Martin, 1984).

[5]Another example is the photons which have two degree of freedom (transverse waves) and that is why the electric and the magnetic waves are perpendicular in the electromagnetic wave.

3. Basic Concepts in the Electroweak Theory

The Electroweak Theory unifies QED (corresponding to the electromagnetic force), with the weak force[1] by the gauge symmetry group representation,

$$G = SU(2) \times U(1). \tag{3.0.1}$$

The theory was introduced by Sheldon Glashow in (1961), and improved independently by Steven Weinberg in (1967) and Abdus Salam in (1968) to what is now called *Glashow Weinberg Salam (GWS) model*. They were awarded the nobel prize in (1979) "for their contributions in determining the weak bosons masses" (Nobel).

The charges in the theory are related by the Gell-Mann-Nishijima formula

$$Q = T^3 + \frac{Y}{2}, \tag{3.0.2}$$

or in terms of the neutral currents and couplings as

$$j^{em}_\mu = \mathbf{J}^3_\mu + \frac{1}{2} j^Y_\mu, \tag{3.0.3}$$

where Q is the electric charge, Y is the weak hypercharge and T^3 is the third component of the weak isospin, they are also the generators of the three gauge groups $U(1)_{em}$, $SU(2)_L$[2], and $U(1)_Y$. Thus, the symmetry gauge group (3.0.1) can be extend to $SU(2)_L \times U(1)_Y$. Note that $\mathbf{J}^3_\mu$, j^Y_μ and j^{em} belong to the gauge symmetry groups, $SU(2)_L, U(1)_Y$ and $U(1)_{em}$, respectively.

The neutral fields W^3_μ and B_μ are orthogonal and can be expressed in terms of the eigenstate neutral fields A_μ and Z_μ as a rotation with mixing angle θ_W, here it is also called the *Weiberg angle*

$$W^3_\mu = A_\mu \sin\theta_W + Z_\mu \cos\theta_W, \quad B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W. \tag{3.0.4}$$

The relation (3.0.2) can be rewritten in terms of the fields currents as

$$-ig\mathbf{J}^3_\mu W^{3\mu} - i\frac{g'}{2} j^Y_\mu B^\mu = -iej^{em}_\mu A^\mu. \tag{3.0.5}$$

Then

$$\begin{aligned}
&= -ig\mathbf{J}^3_\mu \left[A_\mu \sin\theta_W + Z_\mu \cos\theta_W\right] - i\frac{g'}{2} j^Y_\mu \left[A_\mu \cos\theta_W - Z_\mu \sin\theta_W\right] = -iej^{em}_\mu A^\mu, \\
&= -i\left[g \sin\theta_W \mathbf{J}^3_\mu + g' \cos\theta_W \frac{j^Y_\mu}{2}\right] A^\mu - i\left[g\cos\theta_W \mathbf{J}^3_\mu - g' \sin\theta_W \frac{j^Y_\mu}{2}\right] Z^\mu = -iej^{em}_\mu A^\mu,
\end{aligned}$$

Comparing A_μ coefficients yields

$$(e - g\sin\theta_W) j^{em}_\mu - (g\sin\theta_W - g'\cos\theta_W)\frac{j^Y_\mu}{2} = 0, \tag{3.0.6}$$

[1] Note that the two forces unified in low energy level (in the order of $100\ GeV$)(Martin and Shaw, 1992).

[2] T^3 only couple the left-hand component

This implies that

$$e = g\sin\theta_W, \qquad \text{and} \qquad g\sin\theta_W = g'\cos\theta_W. \tag{3.0.7}$$

Therefore, the electromagnetic coupling relates to the weak couplings by the equalities

$$e = g\sin\theta_W = g'\cos\theta_W. \tag{3.0.8}$$

The fermion spinor ψ is written in terms of two components left (χ_L) and right (ψ_R). χ_L is defined as an iso-doublet

$$\chi_L = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \quad l = e, \mu, \tau, \tag{3.0.9}$$

and ψ_R is defined as an isosinglets

$$\psi_R = l_R^-, \tag{3.0.10}$$

The left and right transformation of the ψ components are

$$\begin{aligned} \chi_L &\to e^{i\alpha(x).\mathbf{T}+i\beta(x)Y}\chi_L, \\ \psi_R &\to e^{i\beta(x)Y}\psi_R, \end{aligned} \tag{3.0.11}$$

where $\beta(x)$ is a function in space and time.

On the other hand, the definition for the quarks is slightly different[3]. Take the up u and down d quarks as an example, their left and right components are

$$\begin{aligned} \chi_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \\ \psi_R &= u_r \text{ or } d_r. \end{aligned} \tag{3.0.12}$$

Note that only left-handed doublets are involved in the weak interactions.

The electron field e can be written in term of its left and right components as

$$e = e_R + e_L, \quad \text{where} \quad e_R = \frac{1}{2}(1+\gamma^5)e, \quad e_L = \frac{1}{2}(1-\gamma^5)e, \tag{3.0.13}$$

and γ^5 standard representation is a 4×4 matrix defined as

$$\gamma^5 = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}.$$

Thus,

$$\bar{e}e = \bar{e}\left[\frac{1}{2}(1+\gamma^5) + \frac{1}{2}(1-\gamma^5)\right]e = \bar{e}_L e_R + \bar{e}_R e_L. \tag{3.0.14}$$

[3] Since neutrinos are massless in the theory while the quarks not and note that neutrinos are purely left-handed.

3.1 The Masses of the Electroweak Gauge Bosons

The invariant Lagrangian describe the gauge bosons is

$$\mathcal{L} = \left[D_\mu \phi\right]^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{3.1.1}$$

where ϕ is the same in (2.1.2) and D_μ is the covariant derivative defined as (McMahon, 2008)

$$D_\mu = \partial_\mu + i\frac{g'}{2}B_\mu + i\frac{g}{2}\boldsymbol{\tau}.\mathbf{W}_\mu, \tag{3.1.2}$$

where $\mathbf{W}_\mu = W_\mu^i = \left(W_\mu^1, W_\mu^2, W_\mu^3\right)$, and W_μ^1, W_μ^2 are the charged fields related as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2), \tag{3.1.3}$$

and W_μ^3 is the neutral field. Using the Higgs mechanism with symmetry breaking ($\mu^2 < 0$ and $\lambda > 0$) it is possible to generate the masses by choosing the vacuum ψ_0 to be the same as (2.3.17). Substituting ψ_0 and D_μ in the mass term in (3.1.1) and using (3.1.3) gives

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left[\begin{pmatrix}0 & v\end{pmatrix}(-i\frac{g'}{2}B_\mu - i\frac{g}{2}\boldsymbol{\tau}.\mathbf{W}_\mu)\right]\left[(i\frac{g'}{2}B^\mu + i\frac{g}{2}\boldsymbol{\tau}.\mathbf{W}^\mu)\begin{pmatrix}0\\ v\end{pmatrix}\right], \\
&= \frac{1}{8}v^2\left[g'^2 B_\mu B^\mu - g'gB_\mu W^{3\mu} - gg'W_\mu^3 B^\mu + 2g^2 W_\mu^+ W^{-\mu} + g^2 W_\mu^3 W^{3\mu}\right], \\
&= \frac{1}{4}g^2 v^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2\left[g^2 W_\mu^3 W^{3\mu} - g'gB_\mu W^{3\mu} - gg'W_\mu^3 B^\mu + g'^2 B_\mu B^\mu\right], \\
&= \frac{1}{4}g^2 v^2 W_\mu^+ W^{-\mu} + \frac{v^2}{8}\begin{pmatrix}W_\mu^3 & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{3\mu}\\ B^\mu\end{pmatrix},
\end{aligned} \tag{3.1.4}$$

Now comparing the results with the corresponding mass leads to

$$M_W^2 W^+ W^- = \frac{1}{4}g^2 v^2 W^+ W^- \Longrightarrow M_W = \frac{1}{2}gv. \tag{3.1.5}$$

The second term can be written as

$$\frac{1}{8}v^2\left[(gW_\mu^3)^2 - 2g'gB_\mu W^{3\mu} + (g'B_\mu)^2\right] = \frac{1}{8}v^2\left[gW_\mu^3 - g'B_\mu\right]^2. \tag{3.1.6}$$

Since the determinate of the 2×2 matrix in (3.1.4) is $\det\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix} = 0$, zero is also an eigenvalue of the matrix, then

$$\frac{1}{8}v^2\left[(gW_\mu^3)^2 - 2g'gB_\mu W^{3\mu} + (g'B_\mu)^2\right] = \frac{1}{8}v^2\left[gW_\mu^3 - g'B_\mu\right]^2 + 0\left[g'W_\mu^3 + gB_\mu\right], \tag{3.1.7}$$

which is the orthogonal component of the previous term. Comparing (3.1.7) with the corresponding mass term we find

$$\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2 = \frac{1}{8}v^2\left[gW_\mu^3 - g'B_\mu\right]^2 + 0\left[g'W_\mu^3 + gB_\mu\right]^2. \tag{3.1.8}$$

Using equation (3.0.8), observe that

$$\frac{g'}{g} = \frac{\sqrt{g^2+g'^2}\sin\theta_W}{\sqrt{g^2+g'^2}\cos\theta_W} = \tan\theta_W. \tag{3.1.9}$$

Therefore,

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \tag{3.1.10}$$

and we can draw the coupling triangle, Figure (3.1).

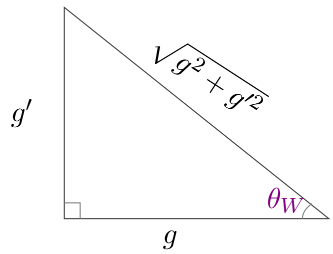


Figure 3.1: The coupling triangle (McMahon, 2008).

The coupling triangle shows that in order for the fields Z_μ and W_μ to couple the mixing angle θ_W should be between zero and $90°$ $(0 < |\theta_W| < 90°)$. Substituting equation (3.1.9) in equation (3.1.8)

$$\begin{aligned}
&\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2 \\
&= \frac{1}{8}v^2\left[gW_\mu^3 - g'B_\mu\right]^2 + 0\left[g'W_\mu^3 + gB_\mu\right], \\
&= \frac{1}{8}v^2(g^2+g'^2)\left[\cos\theta_W W_\mu^3 - \sin\theta_W B_\mu\right]^2 + 0\left[\sin\theta_W W_\mu^3 + \cos\theta_W B_\mu\right].
\end{aligned} \tag{3.1.11}$$

Therefore, the neutral fields are

$$Z_\mu = \left[W_\mu^3\cos\theta_W - B_\mu\sin\theta_W\right], \quad A_\mu = \left[W_\mu^3\sin\theta_W + B_\mu\cos\theta_W\right], \tag{3.1.12}$$

with masses

$$M_Z = \frac{1}{2}v\sqrt{g^2+g'^2}, \quad \text{and} \quad M_A = 0. \tag{3.1.13}$$

Observe that the masses of the W and the Z bosons are not equal $(\frac{M_W}{M_Z} = \cos\theta_W \neq 1)$ and that is an expectable result due the fact that the neutral fields W_μ^3 and B_μ interact with each other[4] via the mixing (coupling) angle θ_W while it does not couple to the charged fields.

Fortunately, the masses of the fermions (quarks and leptons) can be generated using the same the Higgs doublet used in (2.3.18) (Halzen and Martin, 1984).

[4]The mixed term in equation (3.1.6).

3.1.1 The Theoretical and Experimental Mass. GWS predicted the masses of the gauge bosons using the conservation of charge in neutron beta decay

$$n \to p + e^- + \bar{\nu}_e, \tag{3.1.14}$$

or on the quark level

$$(ud)d \to (uu)d + e^- + \bar{\nu}_e, \tag{3.1.15}$$

which is described by the Feynman diagrams

(a) Neutron beta decay $n \to p + e^- + \bar{\nu}_e$.

(b) Neutron beta decay $udd \to uud + e^- + \bar{\nu}_e$.

Figure 3.2: Neutron beta decay (Martin and Shaw, 1992).

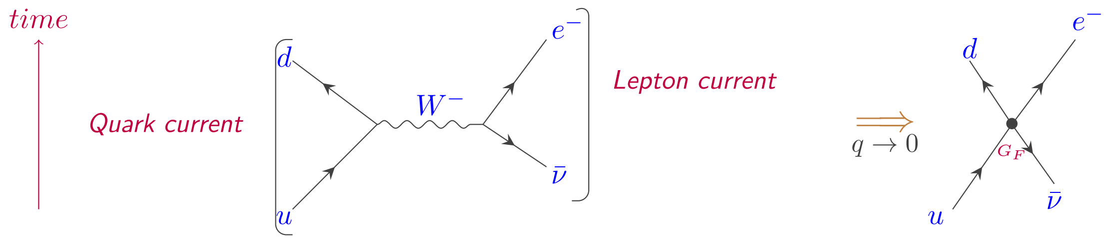
where the change from the neutron's up quark u to the down quark d is accompanied with emitting a virtual W^- boson with charge ($Q_{W^-} = -1$) to conserve the charge[5]. This decay is a low energy weak interaction. GWS has W-propagator

$$\frac{1}{q^2 - M_W^2}, \tag{3.1.16}$$

where $q^\mu = (p^\mu_{neu} - p^\mu_{pro})$. But since $q^2 \ll M_W^2$

$$\frac{1}{q^2 - M_W^2} \approx \frac{1}{-M_W^2}. \tag{3.1.17}$$

The propagator couple quark current to lepton current,



this zero-range point is the Fermi coupling constant, $G_F = 1.166 \times 10^{-5} GeV^{-2}$, with electric charge coupling ($e^2 = 4\pi\alpha$) , where α is the electromagnetic fine-structure constant and has the value $(\frac{1}{137})$ (Griffiths, 2008; Martin and Shaw, 1992).

The vacuum expectation value v is given in term of G_F as (Plehn and Rauch, 2005)

$$v = \left(\frac{1}{\sqrt{2}G_F}\right)^{\frac{1}{2}} = \left(\frac{1}{\sqrt{2} \times 1.166 \times 10^{-5} GeV^{-2}}\right)^{\frac{1}{2}} \approx 246\ GeV. \tag{3.1.18}$$

[5] Where $Q_u = \frac{2}{3}$ and $Q_d = \frac{-1}{3}$.

Then, substituting that in (3.1.5) to find M_W

$$\frac{4M_W^2}{g^2} = \frac{1}{\sqrt{2}G_F} \Longrightarrow M_W^2 = \frac{g^2}{4G_F\sqrt{2}}, \tag{3.1.19}$$

and from equation (3.0.8) we find that the theoretical mass of W bosons is

$$M_W = \left(\frac{g^2}{4G_F\sqrt{2}}\right)^{\frac{1}{2}} = \left(\frac{e^2}{4G_F\sqrt{2}\sin^2\theta_W}\right)^{\frac{1}{2}} = \left(\frac{4\pi\alpha}{4G_F\sqrt{2}\sin^2\theta_W}\right)^{\frac{1}{2}} \approx \frac{37.48015}{\sin\theta_W}\ GeV. \tag{3.1.20}$$

Similarly, the theoretical mass of the Z boson is

$$M_Z = \frac{1}{2}v\sqrt{\frac{e^2}{\sin^2\theta_W} + \frac{e^2}{\cos^2\theta_W}} = \frac{1}{2}ve\sqrt{\frac{\cos^2\theta_W + \sin^2\theta_W}{\cos^2\theta_W\sin^2\theta_W}} = \frac{ve}{\sin 2\theta_W} \approx \frac{74.50403}{\sin 2\theta_W}\ GeV. \tag{3.1.21}$$

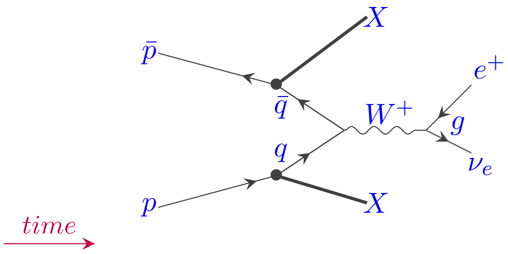
The $W^\pm$ and Z bosons were detected in the proton-anti-proton collision experiment at CERN in (1983) in the reactions

$$\bar{p} + p \to W^+ + X \longrightarrow e^+ + \nu_e, \tag{3.1.22}$$

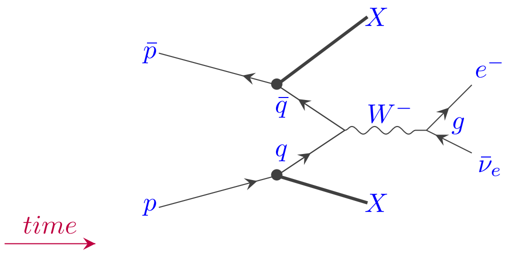
$$\bar{p} + p \to W^- + X \longrightarrow e^- + \bar{\nu}_e, \tag{3.1.23}$$

$$\bar{p} + p \to Z^0 + X \longrightarrow e^+ + e^-. \tag{3.1.24}$$

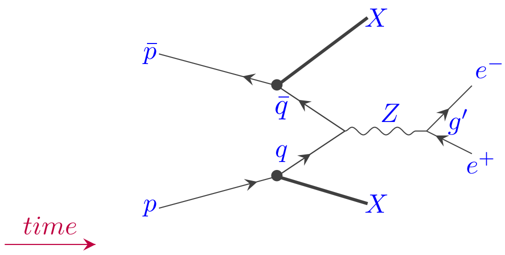
which are described by Feynman diagrams in Figure (3.3)



(a) $\bar{p} + p \to W^+ + X \longrightarrow e^+ + \nu_e$.



(b) $\bar{p} + p \to W^- + X \longrightarrow e^- + \bar{\nu}_e$.



(c) $\bar{p} + p \to Z^0 + X \longrightarrow e^+ + e^-$.

Figure 3.3: Proton-anti-proton collision (Martin and Shaw, 1992)

The transition amplitude involves the propagators $\frac{1}{s-M_W^2}$, and $\frac{1}{s-M_Z^2}$, where $s = (p + \vec{p})^2$ is the total momentum of the initial system. As a function of s, the cross-section have peaks from which the masses are identified to be

$$M_Z = 93 \pm 2\ GeV, \quad M_W = 81 \pm 2\ GeV. \tag{3.1.25}$$

With mixing angle $\theta_W = 28.75^\circ$, substituting it in (3.1.19) and (3.1.20) yields

$$M_Z \approx 91.50696\ GeV, \quad M_W \approx 81.47512\ GeV. \tag{3.1.26}$$

This shows a remarkable agreement between the two results. (Halzen and Martin, 1984; Griffiths, 2008)

3.2 Masses of Leptons and Quarks

3.2.1 Leptons mass. The invariant Lagrangian describing the lepton interactions is

$$\mathcal{L} = \bar{\chi}_L\gamma^\mu\left[\partial_\mu + ig'\frac{Y_L}{2}B_\mu + ig\frac{1}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\right]\chi_L + \bar{\psi}_R\gamma^\mu\left[\partial_\mu - ig'\frac{Y_R}{2}B_\mu\right]\psi_R, \tag{3.2.1}$$

where there is no explicit mass term $(-ml\bar{l})$ because it would spoil $SU(2)_L$ symmetry, since $\bar{l}l = \bar{l}_Ll_R + \bar{l}_Rl_L$. We can generate their masses using the Higgs mechanism.

If we take the electron-neutron lepton pair for instance, where $Y_L = -1$ and $Y_R = -2$, we add an invariant term (under $SU(2)_L \times U(1)_R$) called *Yukawa term*, which couple the Higgs field to the spinor (3.0.9) and (3.0.10)

$$\mathcal{L}_{Yukawa} = -G_e\left[\begin{pmatrix}\bar{\nu}_e & \bar{e}\end{pmatrix}_L\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}e_R + \bar{e}_R\begin{pmatrix}\phi^- & \phi^0\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right], \tag{3.2.2}$$

where G_e is the *Yukawa coupling constant*. Then equation (3.2.1) becomes

$$\begin{aligned}\mathcal{L}_{Yukawa} = {}& \bar{\chi}_L\gamma^\mu\left[\partial_\mu + ig'\frac{Y_L}{2}B_\mu + ig\frac{1}{2}\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\right]\chi_L + \bar{\psi}_R\gamma^\mu\left[\partial_\mu - ig'\frac{Y_R}{2}B_\mu\right]\psi_R \\ & - G_e\left[\begin{pmatrix}\bar{\nu}_e & \bar{e}\end{pmatrix}_L\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}e_R + \bar{e}_R\begin{pmatrix}\phi^- & \phi^0\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right] - \frac{1}{4}\vec{\mathbf{W}}_{\mu\nu}\vec{\mathbf{W}}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},\end{aligned} \tag{3.2.3}$$

Substituting equation (2.3.18) in the mass term, we find

$$\begin{aligned}\mathcal{L}_{Yukawa} &= \frac{G_e}{\sqrt{2}}\left[\begin{pmatrix}\nu_e & \bar{e}\end{pmatrix}_L\begin{pmatrix}0\\ v+h\end{pmatrix}e_R + \bar{e}_R\begin{pmatrix}0 & v+h\end{pmatrix}\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right], \\ &= -\frac{G_e}{\sqrt{2}}\left[(v+h)\bar{e}_Le_R + \bar{e}_R(v+h)e_L\right], \\ &= -\frac{G_e}{\sqrt{2}}v(\bar{e}_Le_R + \bar{e}_Re_L) - \frac{G_e}{\sqrt{2}}h(\bar{e}_Le_R + \bar{e}_Re_L).\end{aligned} \tag{3.2.4}$$

Putting $m_e = \frac{G_e}{\sqrt{2}}v$, which correspond to the mass term in equation (3.2.3)

$$\mathcal{L}_{Yukawa} = -m_e(\bar{e}_Le_R + \bar{e}_Re_L) - \frac{m_e}{v}(\bar{e}_Le_R + \bar{e}_Re_L)h. \tag{3.2.5}$$

Therefore, using equation (3.0.14) we write equation (3.2.5) as

$$\mathcal{L}_{Yukawa} = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh. \tag{3.2.6}$$

Recall that the value of the mass of the electron can not be obtained due to the Yukawa coupling constant G_e being "arbitrary". Since the mass of the electron is "very small", its coupling to the Higgs field $\frac{m_e}{v}$ is thus weak.

The General form of the masses of leptons m_l is

$$m_l = \frac{G_l v}{\sqrt{2}}, \tag{3.2.7}$$

which is directly proportional to G_l, and so stronger the coupling higher the mass and vice versa (Halzen and Martin, 1984).

3.2.2 Quarks mass. We repeat the same steps above, but, in order to generate the mass for all right-handed quarks and maintain the Lagrangian invariance, we will redefine the field ϕ in (2.1.2) as a π rotated 2-axis field. Substituting $\alpha_a(x) = \pi$ in (2.3.1) we get

$$\phi_c = e^{-i\frac{\pi\tau_2}{2}}\bar{\phi} = e^{-i\frac{\pi\tau_2}{2}}\phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix}, \tag{3.2.8}$$

where ϕ^* is the complex conjugate of the field, and $e^{-i\frac{\theta\tau_i}{2}} = (\cos\frac{\theta}{2} - i\tau_i \sin\frac{\theta}{2})$. Note that the ϕ_c transform exactly as ϕ, recalling the transformation (2.3.6)

$$\phi(x) \rightarrow U\phi(x). \tag{3.2.9}$$

We see this by, using equation (2.3.2), (2.3.3) and (2.3.4), and apply it to (3.2.8)

$$-i\tau_2 U^*\phi^* = -i\tau_2 U^* \tau_2\tau_2\phi^* = -iU\tau_2\phi^* = U\phi_c. \tag{3.2.10}$$

In this representation the Higgs field is

$$\phi_c = \frac{1}{\sqrt{2}}\begin{pmatrix} -(v+h) \\ 0 \end{pmatrix}, \tag{3.2.11}$$

and then, in analogy to the lepton Yukawa term in equation (3.2.2) we write the quark Yukawa term

$$\mathcal{L}_{Yukawa} = -G_d \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - G_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} u_R + \text{ hermitian conjugate.} \tag{3.2.12}$$

Breaking the symmetry yields

$$\begin{aligned} \mathcal{L}_{Yukawa} &= -\frac{1}{\sqrt{2}} G_d \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R - \frac{1}{\sqrt{2}} G_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} -(v+h) \\ 0 \end{pmatrix} u_R + \text{ hermitian conjugate,} \\ &= -\frac{G_d}{\sqrt{2}} \left(\bar{d}_L v + \bar{d}_L h\right) d_R - \frac{G_u}{\sqrt{2}} \left(\bar{u}_L v + \bar{u}_L h\right) u_R + \text{ hermitian conjugate,} \\ &= -\frac{G_d v}{\sqrt{2}} \bar{d}d - \frac{G_d}{\sqrt{2}} \bar{d}dh + \frac{G_u v}{\sqrt{2}} \bar{u}u + \frac{G_u}{\sqrt{2}} \bar{u}uh. \end{aligned} \tag{3.2.13}$$

Using $m_d = \frac{G_d}{\sqrt{2}} v$ and $m_u = -\frac{G_u}{\sqrt{2}} v$ we get

$$\mathcal{L}_{Yukawa} = -m_d\bar{d}d - \frac{m_d}{v}\bar{d}dh - m_u\bar{u}u - \frac{m_u}{v}\bar{u}uh. \tag{3.2.14}$$

We can generalize the Lagrangian(3.2.14) to consider all the quarks doublets as

$$\begin{aligned}\mathcal{L}_{Yukawa} &= -G_1^{ij}\bar{L}_i\phi R_j - G_2^{ij}\bar{L}_i\phi^* R_j + \text{ hermitian conjugate},\\ &= m_1^i\bar{L}_iR_j\left(1+\frac{h}{v}\right) - m_2^i\bar{L}_iR_j\left(1+\frac{h}{v}\right), \qquad \text{where } i,j = 1\ldots N.\end{aligned} \tag{3.2.15}$$

We conclude that, none of the masses parameters is obtained theoretically. But, an experiment is made to find the values of those particles including the Higgs boson (Halzen and Martin, 1984). We will briefly talk about the discovery of the Higgs boson in the next chapter.

4. The Discovery of the Higgs Boson

It was discovered in the Light Hadron Collider (LHC), which is the largest particle accelerator in the world. It was built to verify particle physics theories, mainly, the existence of the Higgs boson beside other "unsolved questions" which will improve our understanding of the universe.

The idea is to collide two beams of accelerated protons[1] and study the fragments that result from the collision. The beams were accelerated to very high speed (almost close to the speed of light).

To reach that high speed, firstly, the beam is accelerated by applying electric fields in a linear accelerator called Linac 2 to one-third the speed of light. Secondly, it is accelerated in the Proton Synchrotron Booster, which is a circular accelerator with radius of 25 m , to accelerate the beam to 91.6% of the speed of light. Then, it is sent to the circular accelerator Proton Synchrotron (PS) with radius of 99.9493 m, where it is accelerate to 99.9% of the speed of light, and since the protons approach the speed of light any added energy will transform into mass[2], the protons accelerate until they reached the mass of $25\ GeV/proton$[3] .Then they are "channelled" to the Super Proton Synchrotron (SPS), an accelerator with 8 km circumference, which accelerate the protons to $450\ GeV$. Now the beam is energetic enough to move to the final stage before the collision, which is the LHC with "72 km circumference, with 175 metres depth beneath the Franco-Swiss border near Geneva, Switzerland", Figures (4.1), (4.2). (Lefevre, 2009)

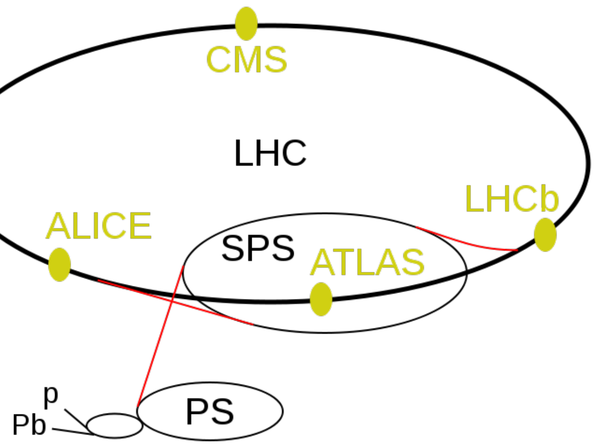


Figure 4.1: The Large Hadron Collider (LHC)

[1]The protons comes from taking off electrons from high compressed hydrogen gas (Lefevre, 2009).
[2]Therefore the protons became heavier as they gain more energy.
[3]Electron volt eV is a unit of energy and it is also use for mass as eV/c^2 but we will use eV for both energy and mass.

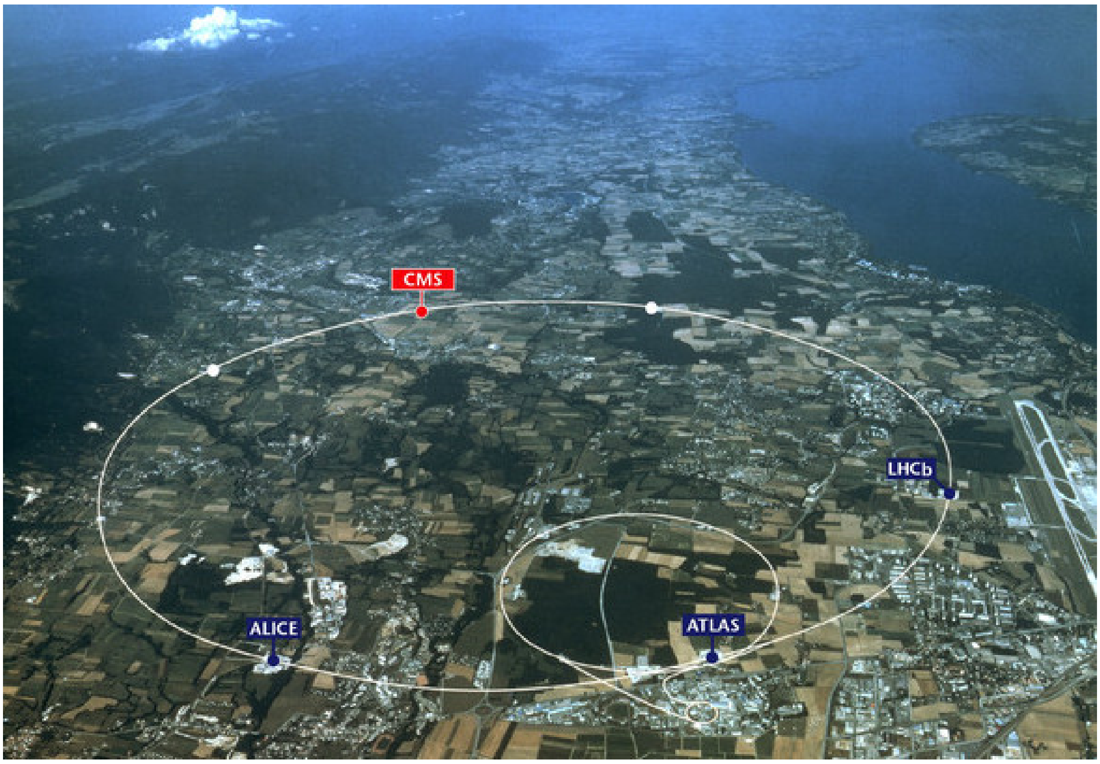


Figure 4.2: The Large Hadron Collider (LargeHadron)

As the beam sent to the LHC is split into two beams and accelerate in opposite direction to reach a mass (energy) of $7\ TeV/proton$[4]. They collide in four points shown in the above figure, which are the collision detectors: A Large Ion Collider Experiment (ALICE), A Toroidal LHC Apparatus (ATLAS), the Compact Muon Solenoid (CMS) and the LHC-beauty (LHC-b). The detectors ATLAS and CMS are the largest among the four (LHC; TheEnd).

The Higgs boson decays very fast and so it can not be detected directly. Instead of that the tracks of its decay products are snapped by the super fast cameras in both detectors, and the data of the decay is collected to analysis by the Worldwide LHC Computing Grid, which is the largest in the world, and then, by using very smart algorithms the decay processes are reconstructed and then compared with the theoretically most probably decay channels of the it.

Since the 1990s many experiments were made to find the Higgs bosons, but they failed in that due to their small collision energy. On the other hand, they limited the range of the search for it which is indicated to be between $(115 - 140)\ GeV$.

If we take the four-lepton decay channel

$$H \to ZZ \to 4l, \tag{4.0.1}$$

to test the existence of the Higgs boson. The two large colliders ATLAS and CMS[5] collected the data and analysed them, exuberance events were observed in the invariant mass range $(125 - 126)\ GeV$ associated with the Z decay in both detectors.The discovery of the Higgs boson was reported in July

[4] Therefore,the total energy of $p\bar{p}$ collision is $14\ TeV/proton$.

[5] They are shut off now, and they will restart in 2015 (TheEnd).

2012, and the masses detected were $126.5\ GeV$ and $125.3 \pm 0.6\ GeV$, in both detectors, respectively. Figure (4.3), Figure (4.4), Figure (4.5) and Figure (4.6) shows the plotted data, and the p-value[6] for mass which shows is clearly very high in the mass range $(125 - 126)\ GeV$.(Higgs boson; TheEnd; Chatrchyan et al., 2012)

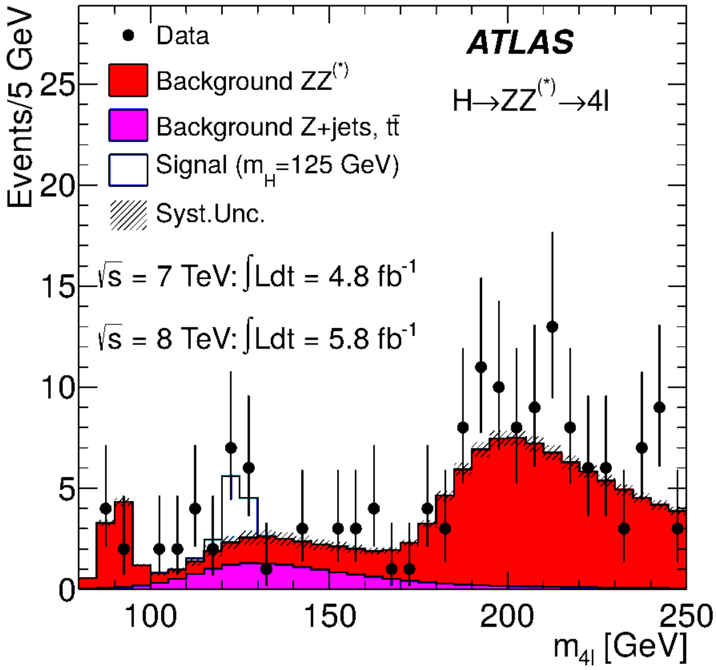


Figure 4.3: ATLAS invariant mass (Aad et al., 2012).

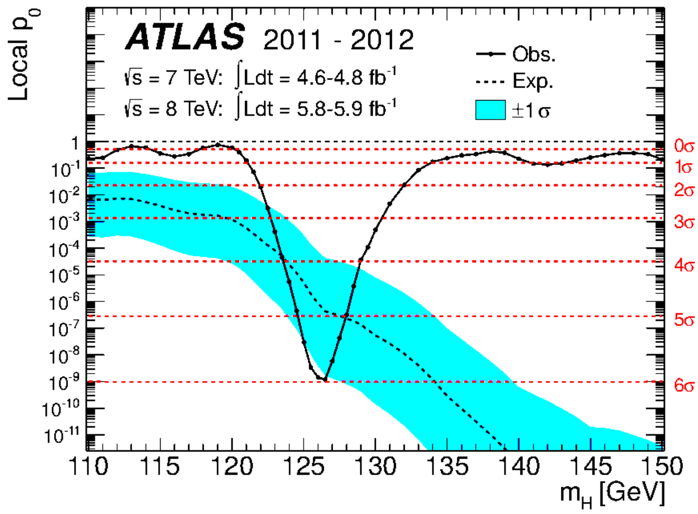


Figure 4.4: ATLAS p-value(Aad et al., 2012).

[6]The p-value measure the accuracy of the measurement.

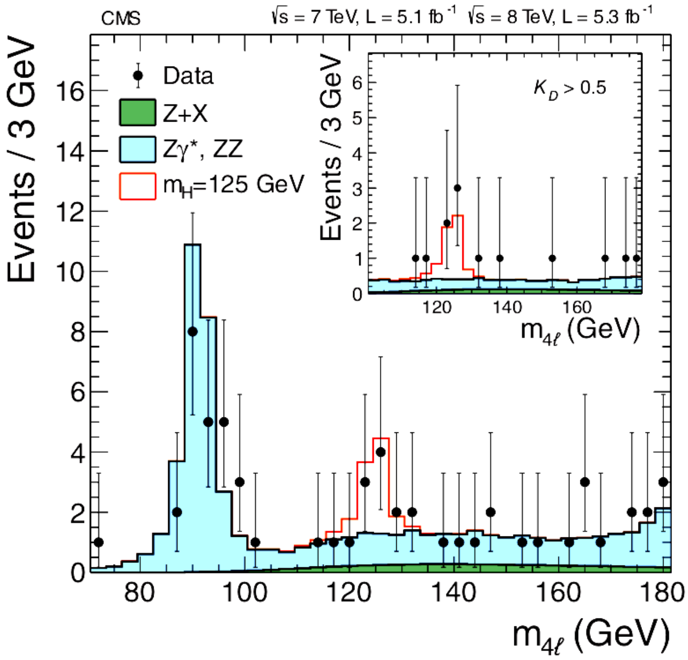


Figure 4.5: CMS invariant mass (collaboration et al., 2012).

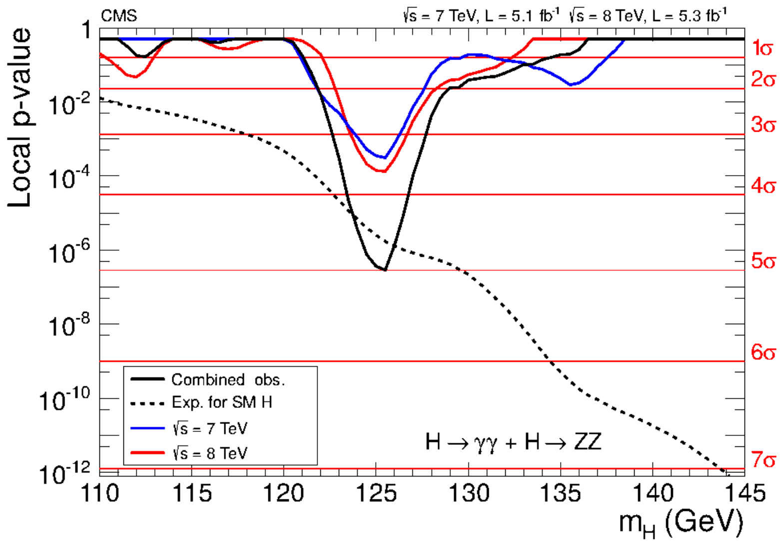


Figure 4.6: CMS p-value (collaboration et al., 2012).

5. Conclusion

We saw how some of the Standard Model particles (bosons and fermions), including the fourth gauge boson included were generated in the GWS model using the spontaneous symmetry breaking idea, and the Higgs mechanism. However, the discovery of the Higgs boson at Cern is considered to be complementary to the Standard Model. The Standard Model was thought to be the perfect model to describe elementary particles, and therefore, our universe. Unfortunately, it is not complete, since a lot of the physical phenomena must still be incorporated into it, such as the gravitational force, the masses of the neutrinos, dark matter and dark energy, which they are still open for both theoretical and experimental physicists, in order to improve our understanding of our universe.

Appendix A. Details of the Calculations

A.1 The Details of the Calculation in Equation (1.0.9)

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial A_\mu} - \frac{\partial}{\partial x^\nu}\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0,\\
&-\frac{1}{2}\kappa^2 A^\mu - \partial_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) = 0,\\
&\partial_\nu F^{\mu\nu} = -\frac{1}{2}\kappa^2 A^\mu,\\
&\partial_\nu\partial^\mu A^\nu - \partial_\nu\partial^\nu A^\mu = -\frac{1}{2}\kappa^2 A^\mu.
\end{aligned} \tag{A.1.1}$$

Using the Coulomb gauge $\partial_\nu A^\nu = 0$ in the first term, we have

$$\partial_\nu\partial^\nu A^\mu = \frac{1}{2}\kappa^2 A^\mu. \tag{A.1.2}$$

And by using the 4-momentum tensor P^μ we find

$$P^\mu P_\mu = (-E^2 + \boldsymbol{p}^2)A^\mu = -m^2 A^\mu = \frac{1}{2}\kappa^2 A^\mu. \tag{A.1.3}$$

A.2 The Details of the Calculation in Equation (2.1.9)

$$\begin{aligned}
\mathcal{L} =& \frac{1}{2}\partial_\mu\xi_1\partial^\mu\xi_1 + \frac{1}{2}\partial_\mu\xi_2\partial^\mu\xi_2 + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu\xi_4\partial^\mu\xi_4 - \frac{1}{2}h^2\mu^2 - h\mu^2 v - \frac{1}{2}\mu^2 v^2 - \frac{1}{2}\mu^2\xi_1^2 - \frac{1}{2}\mu^2\xi_2^2 - \frac{1}{2}\mu^2\xi_4^2\\
&- \frac{1}{4}h^4\lambda - h^3\lambda v - \frac{3}{2}h^2\lambda v^2 - h\lambda v^3 - \frac{1}{4}\lambda v^4 - \frac{1}{2}h^2\lambda\xi_1^2 - h\lambda v\xi_1^2 - \frac{1}{2}\lambda v^2\xi_1^2 - \frac{1}{4}\lambda\xi_1^4\\
&- \frac{1}{2}h^2\lambda\xi_2^2 - h\lambda v\xi_2^2 - \frac{1}{2}\lambda v^2\xi_2^2 - \frac{1}{2}\lambda\xi_2^2\xi_4^2 - \frac{1}{4}\lambda\xi_4^4 - \frac{1}{4}\lambda\xi_2^4 - \frac{1}{2}\lambda\xi_1^2\xi_2^2 - \frac{1}{2}h^2\lambda\xi_4^2\\
&- h\lambda v\xi_4^2 - \frac{1}{2}\lambda v^2\xi_4^2 - \frac{1}{2}\lambda\xi_1^2\xi_4^2.
\end{aligned} \tag{A.2.1}$$

and since v is constant, its derivative is zero and we will drop all the constant terms since they only represent a constant shift in the field, and so Using that $\mu^2 = -\lambda v^2$ we have

$$\begin{aligned}
\mathcal{L} =& \frac{1}{2}\partial_\mu\xi_1\partial^\mu\xi_1 + \frac{1}{2}\partial_\mu\xi_2\partial^\mu\xi_2 + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\partial_\mu\xi_4\partial^\mu\xi_4 + h^2\mu^2 - h\mu^2 v - h^3\lambda v - \lambda v h\xi_1^2 - \lambda v h\xi_2^2\\
&- \lambda v h^3 - \lambda v h\xi_4^2 - \frac{1}{2}\lambda h^2\xi_1^2 - \frac{1}{2}\lambda h^2\xi_2^2 - \frac{1}{2}\lambda h^2\xi_4^2 - \frac{1}{4}\lambda\xi_1^4 - \frac{1}{4}\lambda\xi_2^4 - \frac{1}{4}\lambda h^4 - \frac{1}{4}\lambda\xi_4^4\\
&- \frac{1}{2}\lambda\xi_1^2\xi_2^2 - \frac{1}{2}\lambda\xi_1^2\xi_4^2 - \frac{1}{2}\lambda\xi_2^2\xi_4^2.
\end{aligned} \tag{A.2.2}$$

A.3 Checking the Field Tensor Transform Invariantly in equation (2.3.15)

$$\begin{aligned}
\vec{\mathbf{W}}_{\mu\nu} &= \partial_\mu(U(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu + i\partial_\nu)U^\dagger) - \partial_\nu(U(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu + i\partial_\mu)U^\dagger) - ig\left[(U(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu + i\partial_\nu)U^\dagger), (U(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu + i\partial_\mu)U^\dagger)\right], \\
&= \partial_\mu U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger + i\partial_\mu U\partial_\nu U^\dagger + U\partial_\mu\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu U^\dagger + iU\partial_\mu\partial_\nu U^\dagger + U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu\partial_\mu U^\dagger - iU\partial_\nu\partial_\mu U^\dagger \\
&- \partial_\nu U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger - i\partial_\nu U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger - U\partial_\nu\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger - iU\partial_\mu\partial_\nu U^\dagger - U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\partial_\nu U^\dagger - iU\partial_\nu\partial_\mu U^\dagger \\
&- i[U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger, U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu U^\dagger] - i[U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger, iU\partial_\nu U^\dagger] - i[iU\partial_\mu U^\dagger, U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu U^\dagger] - i[iU\partial_\mu U^\dagger, iU\partial_\nu U^\dagger], \\
&= U\left(\partial_\mu\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu - \partial_\nu\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu - ig[\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu, \vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu]\right)U^\dagger + \partial_\mu Ug(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu + i\partial_\nu)U^\dagger - \partial_\nu g(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu + i\partial_\nu) \\
&+ Ug\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu\partial_\mu U^\dagger - Ug\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu\partial_\nu U^\dagger - U\partial_\nu U^\dagger g(U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu U^\dagger + U\partial_\mu U^\dagger) + U\partial_\mu U^\dagger g(U\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu U^\dagger + iU\partial_\nu U^\dagger) \\
&- Ug\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu UU^\dagger\partial^\mu U^\dagger + Ug\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu UU^\dagger\partial^\nu U^\dagger. \qquad \text{(A.3.1)}
\end{aligned}$$

Using (2.3.7) and (2.3.8) we find

$$\tilde{\vec{\mathbf{W}}}_{\mu\nu} = U\left(\partial_\mu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu) - \partial_\nu(\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu) - ig[\vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\mu, \vec{\boldsymbol{\tau}}.\vec{\mathbf{W}}_\nu]\right)U^\dagger = U\vec{\mathbf{W}}_{\mu\nu}U^\dagger. \qquad \text{(A.3.2)}$$

Which transform invariantly in the Lagrangian since

$$\begin{aligned}
&\frac{1}{4}\tilde{\mathbf{W}^{\mu\nu}}\tilde{\mathbf{W}}_{\mu\nu} = \frac{1}{4}(U\mathbf{W}^{\mu\nu}U^\dagger U\vec{\mathbf{W}}_{\mu\nu}U^\dagger), \\
&= \frac{1}{4}U\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}U^\dagger = UU^\dagger\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu} = \frac{1}{4}\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}. \qquad \text{(A.3.3)}
\end{aligned}$$

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