The Anomalous Electron Magnetic Moment

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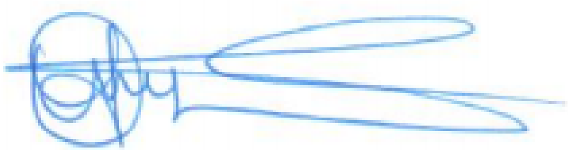
Abstract

In particle physics, subatomic particles such as electrons and muons are assumed to be point-like due to their tiny nature, they posses charge and an intrinsic angular momenta (spin) that allows them to interact with a background of an electromagnetic field. The magnetic moment and the spin are related by a constant, the gyromagnetic ratio g, whose value was predicted by Dirac theory to be *2*. However, experiments and quantum electrodynamic (QED) studies have revealed a slight deviation from this value. This essay focusses on the calculations leading to this anomaly through the use of Feynman diagrams. What is more, we discuss some of the applications of electron's magnetic moment and spin in technology.

Main words: Electron spin, electromagnetic field, magnetic moment, gyromagnetic ratio, Dirac theory, QED, Feynman diagrams.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.



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1. Introduction

The study and measurement of anomalous magnetic moments have provided great insights into the field of subatomic particle physics, the measurement of the proton magnetic moment being one of the first indications of the substructure in the particle (Earl, 2003).

The determination of the anomalous magnetic moment of the electron provides a good way to determine the coupling constant α at $q^2 = 0$, in quantum electrodynamics. Its calculation by Schwinger, Feynman and Tomonaga in 1948, and its agreement with data, was a triumph of quantum field theory (Schwartz, 2013). The calculations discussed in this report provide a unique test of the electroweak theory in the standard model of particle physics. The basic concepts and the Dirac equation are discussed in chapters one and two respectively.

In addition, this essay presents a theoretical approach to the determination of the anomalous magnetic moment of the electron as discussed in chapter three. Here, we calculate the magnetic moment by the help of Feynman rules discussed in the same chapter. In conjunction with the existing theory (Griffiths, 2008), these calculations provide an accurate value of the fine structure constant and hence the value of gyromagnetic ratio (*the ratio of the magnetic moment to the spin of electron*), g. Some of the applications of electron's magnetic moment and intrinsic spin are discussed in chapter four followed by a brief conclusion in chapter five.

The work discussed here can also be extended to give a better understanding of the CPT (charge, parity time reversal) symmetry as well as Lorentz invariance for leptons in general (Barr, 2014).

1.1 Magnetic moment

In this section, we basically define the meaning of "anomalous magnetic moment", describe briefly the reason behind its measurements and explain the general ideas involved. We will start by explaining the meaning of the following terms (Gross, 2008):

- Nuclear spin:- The total intrinsic angular momentum of the nucleus in an atom.
- Gyromagnetic ratio:- The ratio of the magnetic moment to the angular momentum.
- Bohr's magneton:- This is the unit of atomic magnetic moment.
- Lande's g-factor:- The units of magnetic moment in Bohr's magneton

Definition: Magnetic dipole moment, $\boldsymbol{\mu}$ is the measure of how much torque, $\boldsymbol{\tau}$ an object experiences in a magnetic field, mathematically expressed as

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}.$$

The potential energy associated with this force is given as

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}.$$

For the subatomic particles such as electrons, muons and taus, the magnetic moment is brought about by an intrinsic spin, $\mathbf{S}$ and the relationship between the two quantities is brought about by a constant known as the gyromagnetic ratio g defined above. Mathematically, $\boldsymbol{\mu}$ and g are related as follows:

$$\boldsymbol{\mu} = g\left(\frac{e}{2m}\right)\mathbf{S}. \tag{1.1.1}$$

For point-like particles, the Dirac theory predicted the value of g to be exactly 2 $(g = 2)$ - *this will be revised in section 2.3.1*. However, experiments have revealed a value that is slightly greater than 2 for charged leptons. This controversy can only be well understood in quantum field theory. Studies in quantum field theory have shown that this deviation is as a result of higher order interactions, for point-particles, and rich internal structures for composite particles like protons and neutrons.

The table below shows some data on deviation of the $g-$ value from $g = 2$ for various subatomic particles (Earl, 2003)

Particle	Experimental value	Theoretical prediction	Relative precission
Electron	2.0023193043738	2.00231930492	4×10^{-12}
Muon	2.0023318406	2.0023318338	8×10^{-10}
Tau	2.008	2.0023546	4×10^{-2}

Table 1.1: *Comparison of theoretical and experimental g-values of various subatomic particles.*

From the table above, we notice that the gyromagnetic ratios of stable and nearly stable particles can be measured to very high precisions. In addition, leptons have g-values that can be calculated very precisely in the context of the standard model and the above comparisons gives an important test for theory (Earl, 2003). These comparisons can also help us to test quantum electrodynamics (QED), the theory that is generally considered the most accurate physical theory. Furthermore, we can also introduce Bohr's magneton via the magnetic moment as follows (Earl, 2003)

$$\boldsymbol{\mu} = g\left(\frac{e\hbar}{2m}\right)\frac{\mathbf{S}}{\hbar}$$

where $\hbar$ is the Planck's constant , m is the mass of the particle and the ratio $\dfrac{e\hbar}{2m}$ is the Bohr's magneton.

Classical electromagnetism predicts $g = 1$ if the particle has identical mass and charge distributions. In this case, we assume that the electron's spin is a rotation about its axis of a radius $\mathbf{r}$. If the velocity of the electron is $\mathbf{v}$, the current of density $\mathbf{J}$ flows, and we have:

The current (density):

$$\mathbf{J} = e\mathbf{v}.$$

The magnetic moment:

$$\boldsymbol{\mu} = \frac{1}{2}(\mathbf{r} \times \mathbf{J}) = \frac{e}{2}(\mathbf{r} \times \mathbf{v}) = \frac{e}{2m}(\mathbf{r} \times \mathbf{p}),$$

$$\boldsymbol{\mu} = \frac{e}{2m}\mathbf{L}.$$

Comparing this with equation (1.1.1) above, we readily note that $g = 1$ and this implies that classical prediction completely disagrees with the experimental results.

1.2 The standard model in a nutshell

According to Best (2014), the universe is basically made up of two major components: matter and energy.

Matter: It makes up 26% of the universe, 4% of this is in atoms (3.6% of which is intergalactic gas) while 22% of this composition is dark matter.

Energy: The remaining 74% of the universe volume is dark energy. The gradual expansion of the universe is believed to be as a result of repulsive forces resulting from the dark energy.

From Einstein's equation, $E = mc^2$, energy can be converted to matter (mass, m) and vice versa (Best, 2014). This section focuses on the standard model that was set up in 1970's, it describes the universe in terms of matter (basically fermions) and force (basically bosons). In this model, we use 17 fundamental particles to describe the other 200 particles of the periodic table. These particles are fermions and bosons.

1.2.1 Fermions.

Fermions are leptons and quarks.

Leptons

We basically have three leptons, and three neutrinos thus making a total of six in number. These are given below:

Lepton	Neutrino
Electron(e)	electron neutrino(ν_e)
Muon(μ)	muon neutrino(ν_μ)
Tau(τ)	tau neutrino(ν_τ)

Table 1.2: *The six leptons with their respective symbols.*

Leptons do not reside in the nucleus, and they do not take part in strong interactions. In addition, they lack internal structures and posses a charge of -1 while their neutrino counterparts have no charge.

We also note that all leptons exhibits a spin $\frac{1}{2}$ property and what is more, *electrons* are stable while *neutrinos*, *muons* and *taus* are unstable and decays into leptons and other subatomic particles when isolated (Barr, 2014). For instance;

$$\mu^- \to \overline{\nu}_e + \nu_\mu + e^- \quad (\text{half life} = 2.20 \times 10^{-6} s),$$

$$\tau^- \to \overline{\nu}_\mu + \nu_\tau + \mu^- \quad (\text{half life} = 2.90 \times 10^{-13} s).$$

Even though the decays occur in a weak interaction, not all weak interaction decays lead to formation of neutrinos (Manoukian, 2007).

Quarks

We generally have six quarks which exists in various flavours, each with a charge that is a fraction of that of the electron (Altland, 2012). They are as follows:

Quark flavour	Charge
up(u)	$+\frac{2}{3}$
down(d)	$-\frac{1}{3}$
charm(c)	$+\frac{2}{3}$
strange(s)	$-\frac{1}{3}$
top(t)	$+\frac{2}{3}$
bottom(b)	$-\frac{1}{3}$

Table 1.3: *The six flavours of quarks and their respective charges.*

Quarks can be combined together to form hadrons.This happens in two ways to form the two known types of hadrons:

- Baryons:- These are three-quark hadrons and examples are neutrons (ddu-quarks) and protons (uud-quarks).
- Mesons:- These are made up of a quark and an anti-quark and an example is a pion.

Baryons are confined within the nucleus and some like protons are stable while others like neutrons are unstable. The unstable baryons undergo decay when isolated. For example:

$$\underset{\text{uds}}{\Lambda^\circ} \longrightarrow \underset{\bar{\text{u}}\text{d}}{\pi^-} + \underset{\text{uud}}{p} \, .$$

From this example, a three-quark baryon (lambda) decays into another three quark hadron (proton) and a quark-antiquark pion. However, we notice that the strangeness is not conserved.

In contrast, charged mesons are unstable and decay into electrons and neutrinos, while neutral mesons may decay into photons when isolated. The main difference between the two types of hadrons is that baryons are spin-$\frac{1}{2}$ hadrons while mesons posses integer spins.

1.2.2 Bosons.

Bosons are integer-spin particles and unlike fermions, they do not obey Pauli's exclusion principle i.e. two or more bosons can occupy the same energy state. We generally have two categories of bosons: gauge bosons and the Higgs boson.

Gauge bosons

In the standard model, photons (γ), $W^{\pm}$-bosons, gluons (g) and Z-bosons are all categorised as gauge bosons (Best, 2014). They can be distinguished based on their masses, a property which also determines which type of interactions they are involved in. For instance, photons interacts electromagnetically while $W^{\pm}$-bosons and Z-bosons are involved in weak interactions (larger masses). In addition, $W^{\pm}$ can also interact electromagnetically but to a smaller extent. We can therefore deduce that these gauge-bosons generally play a part in electroweak interactions and their behaviour in interactions can be better understood in Quantum electrodynamic (QED) studies.

There exists a total of *eight gluons* that mediate the strong interactions between quarks and just like photons, gluons have little or no mass and are colour sensitive (Griffiths, 2008), a property upon which their strong interactions and interaction amongst themselves is based. However, their explicit behaviour is described in quantum chromodynamics (QCD) studies which will not be considered here.

Higgs boson

This type of boson was theoretically predicted in 1964 by Peter Higgs and unlike fermions, it has a spin of zero thus classified together with other integer spin particles (Barr, 2014). In the standard model, it is used as a reference particle when describing the masses of all other elementary particles. The Higgs boson can participate in weak interaction, electromagnetic interaction as well as individual interactions hence it is extremely unstable (Best, 2014). Even though experimental attempts to determine its nature have not been successful for sometime, its existence was finally confirmed in March 2013.

1.3 Experimental evolution

QED predicts that the anomalous value of g results from vacuum polarisations and fluctuations. In this theory, the relationship between the fine structure constant α and Lande's g-factor are represented as an asymptotic series expansion from which the anomalous electron magnetic moment can be deduced

$$g = 2\left[1 + k_1\left(\frac{\alpha}{\pi}\right) + k_2\left(\frac{\alpha}{\pi}\right)^2 + k_3\left(\frac{\alpha}{\pi}\right)^3 + \cdots + a_{\mu\tau} + a_{had} + a_{weak}\right].$$

The constant coefficients and the term $a_{\mu\tau}$ have been calculated while the other terms are non-QED polarisations and weak observables (Odom, 2004). The fine structure constant α is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.036}.$$

Several experiments have been done over years beginning 1953, with the universal motive to determine the possible internal structure of an electron and a possible interaction with the vacuum polarisations of QED. These experiments involve use of cyclotrons - *a type of particle accelerator in which charged particles are accelerated outwards by a rapidly changing electric field while being held from the centre along a spiral path by a static magnetic field*, and a penning trap, in which an electron is considered to be a mechanical system with equal distributions of charge and mass (Earl, 2003). The value of g is then calculated directly by use of measured cyclotron frequency and spin frequencies, taking into consideration the possible error sources in these experiments.

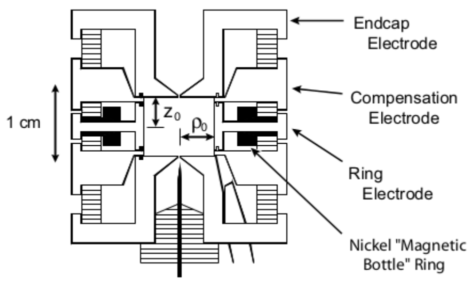


Figure 1.1: The cross-sectional area of the penning trap (Odom, 2004)

The idea of measuring $g - 2$ instead of g was developed in 1953 when the first experimental attempt to determine g-value was made. These series of experiments were done in the University of Michigan

(Odom, 2004). It involved placing a number of electrons in a magnetic mirror trap and then studying the relative orientation of the precessing spin and the orbital angular moment vectors. The result from this experiment revealed the value of g to be $g \approx 2.002319$.

Between 1977 and 1999, other experiments were done on a similar set up at the University of Washington and the outcome was a great improvement in the previous work, the experiment was improvised to detect only single electrons in the penning traps.

In 1987, the improvement in the experiment involved use of molybdenum electrodes in the penning traps, and the outcome was $g \approx 2.002313$. This was a great milestone as it was considered to be a more accurate value. However, it had a limitation of non-reliable data values and inability to achieve finer line widths (Odom, 2004). The same experiments were repeated with a positron and it resulted into a different value of $g \approx 2.002319$. The inaccuracy was as a result of drive-shift and cavity-shift systematics (Odom, 2004).

In the early 1990's, the molybdenum electrodes were replaced with phosphor-bronze electrodes in order to reduce the shifts that were initially a limitation, the modifications yielded a result that could not show a Gaussian distribution when plotted.

The latest experiments were established in Harvard University (2004). The aim is to improve the existing results and also to probe the $g - \text{value}$ of other subatomic particles such as muons. The value from this Harvard experiment (2004) stood at $g = 2.00231930436\ (57)$ (Earl, 2003). This is a better result since it is very accurate, and almost in agreement with the result obtained from the QED calculations.

2. Single particle Dirac equation

As a consequence of the Klein-Gordon theory's failure (Wachter, 2010) to provide an amicable interpretation of the negative energy solutions and to explain the lack of positive definite probability density for the wave-function, Paul Dirac (1928) came up with a relativistic equation that could be used to describe the spin $\frac{1}{2}$ particles and provide a better explanation to the constraints experienced by relativistic Klein-Gordon equation to a greater extent.

In this chapter, we discuss the derivation of the relativistic Dirac equation from the first order time dependent Schrödinger equation and express it in both canonical and Lorentz covariant form. What is more, we solve the equation explicitly and discuss the significance of the spinors in relation to the equation.

2.1 The Dirac equation

The one-particle interpretation of the Klein-Gordon equation was not seriously considered until Dirac introduced the relativistic quantum mechanics for the spin $\frac{1}{2}$ particles (Wachter, 2010). In order to understand this relativistic consideration for the spin $\frac{1}{2}$ particles, the following principles must hold:

- We must obtain the non-relativistic quantum mechanics in the appropriate limit.
- Limitation of one particle interpretation to the small interaction energy in comparison to the rest energy and position uncertainty of the wave function (Hannabuss, 1997).

However, the Dirac equation that we will discuss here also experienced a problem that is universal to every relativistic wave equation, that is, it also gives the negative energy solutions whose physical meaning and/or interpretation is still open for further probe. In this section, we will notice that solutions to Dirac equation exhibit an inner degree of freedom that can be attributed to the spin of the particle $s = \frac{1}{2}$.

2.1.1 Canonical form.

Generally, the relativistic energy-momentum relation for the free particle is given by:

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4. \tag{2.1.1}$$

We assign the natural units

$$\hbar = c = 1,$$

so that we have

$$E^2 = \mathbf{p}^2 + m^2. \tag{2.1.2}$$

From the correspondence principle, the operator replacement for time and position can be done as follows (Kumericki, 2001).

$$E \to i\partial_t, \quad \mathbf{p} \to -i\nabla.$$

If we implement these operators into equation (2.1.1), we get the Klein-Gordon equation which is an equation for a scalar field, expressed as

$$\left[\Box + m^2\right] \psi = 0.$$

This is the equation that experienced the above mentioned constraints. Dirac started by considering a relativistic generalization of the time dependent Schrödinger equation for a free particle:

$$i\partial_t\psi(x) = H\psi(x), \tag{2.1.3}$$

where the energy operator H is hermitian and $x = x^\mu = \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}$.

From equation (2.1.3), he made the following hypotheses:

1. The equation is first order in time and space.
2. The equation should give an energy-momentum relation in operator form.
3. The probability density $\rho = \psi^*\psi$ and the four-current vector j^μ are conserved $(\partial_\mu j^\mu = 0)$ in time and space and related by the continuity equation.

From 1 and 2, the following **ansatz** is sufficient

$$H = \mathbf{p}\cdot\boldsymbol{\alpha} + \beta m \tag{2.1.4}$$

where m is the rest mass of the particle. This Hamiltonian is equivalent to the relativistic energy hence it can also be secondarily expressed as

$$H^2 = \mathbf{p}^2 + m^2$$

or

$$-\frac{\partial^2}{\partial t^2}\psi(x) = (\mathbf{p}^2 + m^2)\psi(x).$$

This is the Klein-Gordon equation. From this equation, we can deduce that α_i and β have a certain algebraic form which we have to find as there are no mixed terms in $\boldsymbol{\alpha}\cdot\mathbf{p}$ and β. According to Wachter (2010), we can proceed in the following manner to determine the nature of these unknowns.

From equation (2.1.3) above, we have

$$i\partial_t\psi = \left(\frac{1}{i}\sum_i \alpha_i\partial_i + \beta m\right)\psi.$$

Multiplying both sides by $i\partial_t$,

$$-\frac{\partial^2\psi}{\partial t^2} = i\partial_t\left(\frac{1}{i}\sum_i \alpha_i\partial_i\psi + \beta m\psi\right)$$

$$= \frac{1}{i}\sum_j \alpha_j\partial_j\left(\frac{1}{i}\sum_i \alpha_i\partial_i\psi + \beta m\psi\right) + \beta m\left(\frac{1}{i}\sum_i \alpha_i\partial_i\psi + \beta m\psi\right),$$

hence

$$-\frac{\partial^2\psi}{\partial t^2} = -\sum_{i,j}\frac{\alpha_i\alpha_j + \alpha_j\alpha_i}{2}\partial_i\partial_j\psi + \frac{m}{i}\sum_i\left(\alpha_i\beta + \beta\alpha_i\right)\partial_i\psi + \beta^2 m^2\psi.$$

Since $\partial_i\partial_j = \partial_j\partial_i$, we clearly see that equation (2.1.4) can only be valid if α_i and β are matrices for which the following conditions hold

$$\alpha_i^2 = \beta^2 = 1,\ \{\alpha_i, \beta\} = 0,\ \{\alpha_i, \alpha_j\} = 2\delta_{ij} \tag{2.1.5}$$

and $\alpha_i = \alpha_i^\dagger$, $\beta = \beta^\dagger$ since H is to be hermitian. Then the eigenvalues of the operator matrices must be ± 1 with vanishing *traces*. The dimension of each matrix is even-numbered with the least even number $n = 2$ leading to the three known Pauli matrices but we need at least four such matrices, hence, we will work with $n = 4$. From the conditions in equation (2.1.5) above, we can get the explicit Dirac representation of the matrices α_i and β (Hannabuss, 1997)

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where σ_i are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

with the identity

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k. \tag{2.1.6}$$

If we take the least appropriate dimension, $n = 4$, we get the free particle Schrödinger equation (2.1.16) transformed into:

$$i\partial_t \psi_i(x) = \sum_{j=1}^{4} \underbrace{\Big((\boldsymbol{\alpha} \cdot \mathbf{p})_{ij} + \beta_{ij} m \Big)}_{H} \psi_j(x), \text{ with } \; i = 1, 2, \cdots 4. \tag{2.1.7}$$

From this expression, we note that the bispinor, $\psi(x)$ must now be a 4×1 matrix. This is the free particle Dirac equation in canonical/ Hamilton form. The Hamiltonian H is hermitian thus we can achieve a positive definite probability density which obeys the continuity equation.

$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0$$

where the current density and the probability density are given by $\mathbf{j} = \psi^\dagger \boldsymbol{\alpha} \psi$ and $\rho = \psi^\dagger \psi$ respectively.

Proof. We now prove that $\partial_t \rho + \nabla \cdot \mathbf{j} = 0$ for $\mathbf{j} = \psi^\dagger \boldsymbol{\alpha} \psi$ and $\rho = \psi^\dagger \psi$ by proceeding as follows:

From equation (2.1.7), we can alternatively write the canonical form of Dirac equation as follows

$$i\partial_t \psi = \big(\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m\big)\psi = H\psi.$$

We find the probability current for this wave function by multiplying the canonical equation by $\psi^\dagger$ from the left and subtracting with its adjoint multiplied by ψ from the right (Potamianos, 2007)

$$\psi^\dagger i\partial_t \psi = \psi^\dagger \big(-i\boldsymbol{\alpha} \cdot \nabla + \beta m \big)\psi,$$

$$-\big(i\partial_t \psi^\dagger\big)\psi = \big(i\nabla\psi^\dagger \cdot \boldsymbol{\alpha} + m\psi^\dagger \beta\big)\psi.$$

Subtracting the two equations results in

$$i\partial_t \psi^\dagger \psi + i\nabla\big(\psi^\dagger \boldsymbol{\alpha} \boldsymbol{\psi}\big) = 0$$

and proves the above statement. □

2.1.2 Lorentz covariance form.

We now show that the Dirac equation has the same form in all inertial systems, based on the earlier stated principles i.e the symmetry between $ct = x^0$ and x^i must be maintained in accordance with the relativity principle.

We therefore introduce the γ-matrices that are defined in Clifford algebra.

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad (\gamma^\mu)^2 = g^{\mu\mu}$$

$$\beta = \gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} = \beta\alpha_i. \tag{2.1.8}$$

Now, since α_i and β are hermitian,

$$\gamma^{\mu\dagger} = g^{\mu\mu}\gamma^\mu \Longleftrightarrow \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$

(Schwartz, 2013).

The interaction with the electromagnetic field is incorporated by minimal substitution

$$p_\mu \to p_\mu - eA_\mu$$

where A^μ is the vector field while

$$\mathbf{E} = -\nabla\phi - \partial_t\mathbf{A} \text{ and } \mathbf{B} = \nabla \times \mathbf{A} \tag{2.1.9}$$

in the normal relativistic representation. We can now rewrite the canonical equation (for a minimally coupled electromagnetic field) as

$$\left[\gamma^\mu(p_\mu - eA_\mu) - m\right]\psi = 0 \tag{2.1.10}$$

where $A_\mu = A_\mu(x)$ and $\psi = \psi(x)$

or,

$$(i\gamma^\mu\partial_\mu - m)\psi = e\not{A}\psi \Longrightarrow (i\not{\partial} - m)\psi = e\not{A}\psi$$

or, for a completely free field (without any electromagnetic coupling),

$$(i\not{\partial} - m)\psi = 0. \tag{2.1.11}$$

In order to have a form invariant Dirac equation, we will consider two inertial frames T_1 and T_2 with observers O_1 and O_2 respectively (Kumericki, 2001) such that, if we have a wave function $\psi(x^\mu)$ describing a particle in T_1 then we can determine a corresponding $\psi'(x'^\mu)$ in T_2 that describes the same particle. Here, if Λ is the transformation of the coordinates, then $x'^\nu = \Lambda^\nu_\mu x^\mu$ while $\psi(x^\mu)$ and $\psi'(x'^\mu)$ are solutions of Dirac equation in T_1 and T_2 respectively;

$$\left(i\gamma^\mu\frac{\partial}{\partial x^\mu} - m\right)\psi(x^\mu) = 0 \tag{2.1.12}$$

and

$$\left(i\gamma^\nu\frac{\partial}{\partial x'^\nu} - m\right)\psi'(x'^\nu) = 0. \tag{2.1.13}$$

If the Dirac equation is Lorentz covariant, then it implies that the γ-matrices are the same in T_1 and T_2 and therefore we proceed by finding a transformation matrix M such that

$$M\psi(x) = \psi'(\Lambda x). \tag{2.1.14}$$

Proof. Suppose we apply the condition in equation (2.1.14) to equation (2.1.12), we obtain the following.

$$iM\gamma^\mu M^{-1}\frac{\partial}{\partial x^\mu}M\psi(x^\mu) - mM\psi(x^\mu) = 0$$

implying that

$$iM\gamma^\mu M^{-1}\frac{\partial}{\partial x^\mu}\psi'(x'^\nu) - m\psi'(x'^\nu) = 0. \tag{2.1.15}$$

Applying the chain rule

$$\frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial x'^\nu}\frac{\partial x'^\nu}{\partial x^\mu} = \Lambda^\nu_\mu\frac{\partial}{\partial x'^\nu}.$$

We therefore have

$$iM\gamma^\mu M^{-1}\Lambda^\nu_\mu\frac{\partial}{\partial x'^\nu}\psi'(x'^\nu) - m\psi'(x'^\nu) = 0,$$

or

$$(iM\gamma^\mu M^{-1}\Lambda^\nu_\mu\frac{\partial}{\partial x'^\nu} - m)\psi'(x'^\nu) = 0. \tag{2.1.16}$$

If we compare equations (2.1.16) and (2.1.13), we notice that

$$\gamma^\nu = M\gamma^\mu M^{-1}\Lambda^\nu_\mu,$$

or more explicitly,

$$M(\Lambda)\gamma^\mu M^{-1}(\Lambda) = (\Lambda^{-1})^\nu_\mu\gamma^\nu. \tag{2.1.17}$$

□

We have shown that the Dirac equation is form invariant under Lorentz transform and therefore we can conclude that it is a relativistic generalisation of Schrödinger equation. The matrix M can be obtained from Λ^ν_μ by iterating the infinitesimal Lorentz transformations.

2.2 Explicit solution

Let us now consider the Hamiltonian in the free Dirac equation (2.1.7) above

$$H = \boldsymbol{\alpha}\cdot\mathbf{p} + \beta m, \quad i\partial_t\psi = H\psi. \tag{2.2.1}$$

We will have an **ansatz** for the solution of the equation, expressed as

$$\psi(x) = \begin{pmatrix}\varphi_0\\ \chi_0\end{pmatrix}e^{i(\mathbf{p}\cdot\mathbf{x}-Et)}.$$

(Wachter, 2010)

In this case, ψ_0 and χ_0 are the component spinors. If we substitute $\psi(x)$ into equation (2.2.1), we have

$$\begin{aligned}(E-m)\varphi_0 - \boldsymbol{\sigma}\cdot\mathbf{p}\chi_0 = 0,\\ -\boldsymbol{\sigma}\cdot\mathbf{p}\varphi_0 + (E+m)\chi_0 = 0.\end{aligned} \tag{2.2.2}$$

Non-trivial solutions exists for this pair of equations only if the coefficient determinants vanishes hence

$$\begin{vmatrix} E-m & -\boldsymbol{\sigma}\mathbf{p} \\ -\boldsymbol{\sigma}\mathbf{p} & E+m \end{vmatrix} = E^2 - m^2 - (\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{p}) = 0.$$

We now use the identity $(\boldsymbol{\sigma}\cdot\mathbf{p})^2 = \mathbf{p}^2$ to obtain

$$E = \pm\sqrt{\mathbf{p}^2 + m^2} \tag{2.2.3}$$

as expected. We now substitute these into equation (2.2.2) which yields

$$\begin{aligned} \psi^+(x) &= \begin{pmatrix} \chi^{(\xi)} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p+m}\chi^{(\xi)} \end{pmatrix} e^{-i(E-\mathbf{p}\cdot\mathbf{x})}, \\ \psi^-(x) &= \begin{pmatrix} \frac{-\boldsymbol{\sigma}\cdot\mathbf{p}}{p+m}\chi^{(\xi)} \\ \chi^{(\xi)} \end{pmatrix} e^{i(E+\mathbf{p}\cdot\mathbf{x})}, \end{aligned} \tag{2.2.4}$$

where, $\xi = (1,2)$, $e^{-i(E-\mathbf{p}.\mathbf{x})} \sim \psi^{(\xi)}_{\mathbf{p}}(x)$ and $e^{i(E+\mathbf{p}\cdot\mathbf{x})} \sim \psi^{(\xi)}_{-\mathbf{p}}(x)$.

Since $\chi^{(\xi)}$ are generally constant component spinors up to normalization, we can reformulate our **ansatz** to read

$$\psi(x) = u(\mathbf{p})e^{-ipx}$$

which, when we include into the Dirac equation, gives the momentum space Dirac equation (Kumericki, 2001).

$$(\not{p} - m)u(\mathbf{p}) = 0.$$

We now have

$$\begin{aligned} u_+(\mathbf{p},\xi) &= N\begin{pmatrix} \chi^{(\xi)} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi^{(\xi)} \end{pmatrix}, \\ u_-(\mathbf{p},\xi) &= -N\begin{pmatrix} \frac{-\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi^{(\xi)} \\ \chi^{(\xi)} \end{pmatrix}. \end{aligned} \tag{2.2.5}$$

where $N = \sqrt{\frac{E+m}{2E}}$ is the normalization constant which can be calculated using the identity $u^\dagger u = 1$ $\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the spinors.

Note: For our case, we will only use the positive energy for the free electron in a momentum space, i.e.

$$u_+(\mathbf{p},\xi) = N\begin{pmatrix} \chi^{(\xi)} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi^{(\xi)} \end{pmatrix}. \tag{2.2.6}$$

2.3 Coupling of magnetic field and Pauli Hamiltonian

A charged electron e in an electromagnetic field experiences a Lorentz force expressed as

$$\mathbf{F} = e\big(\mathbf{E} + \mathbf{v} \times \mathbf{B}\big),$$

where $\mathbf{v}$ is the velocity of the electron. If a vector potential $\mathbf{A}(x)$ and a Coulomb potential $\phi(x)$ are the corresponding potentials to electric and magnetic field respectively, then the conditions in 2.1.9 holds. Classically, it can be proved (Greiner, 2001), (pg.213-214) that this motion is described by a Hamiltonian function given as

$$H = \frac{1}{2m}\big(\mathbf{p} - e\mathbf{A}\big)^2 + e\phi. \tag{2.3.1}$$

The terms $\mathbf{p} - e\mathbf{A}$ and $e\phi$ are the minimum coupling while the canonical momentum $\mathbf{p}$ can be expressed as,

$$\mathbf{p} = m\mathbf{v} + e\mathbf{A}$$

and is determined only by the vector potential. Using the condition that $p \to -i\nabla$ in a quantum field, we can transform the classical Hamiltonian function into an operator

$$\hat{H} = \frac{1}{2m}\big(-i\nabla - e\mathbf{A}\big)^2 + e\phi. \tag{2.3.2}$$

Noting that gradient and vector potentials do not commute, we can expand the Hamiltonian Greiner (2001) to obtain

$$\hat{H} = -\frac{1}{2m}\nabla^2 + \frac{ie}{m}(\mathbf{A}\cdot\nabla) + \frac{ie}{2m}(\nabla\cdot\mathbf{A}) + \frac{e^2}{2m}\mathbf{A}^2 + e\phi.$$

Since $\nabla \cdot \mathbf{A} = 0$ ($\mathbf{A}$ and ϕ are not unique but Gauge dependent-*studied in Gauge theory not discussed here*). Rearranging the terms and using the momentum operator $\hat{p}$, we have

$$\hat{H} = \frac{\hat{p}^2}{2m} + e\phi - \frac{e}{m}\mathbf{A}\cdot\hat{p} + \frac{e^2}{2m}\mathbf{A}^2,$$

or

$$\hat{H} = \hat{H}_0 - \frac{e}{m}\mathbf{A}\cdot\hat{p} + \frac{e^2}{2m}\mathbf{A}^2. \tag{2.3.3}$$

The operator $\hat{H}_0$ represents the motion in an uncoupled field while $(\mathbf{A}\cdot\hat{p})$ represents the motion in a coupled field. Moreover, the third term only depends on the vector potential $\mathbf{A}$ and therefore can be dropped when the field is weak (Manoukian, 2007). Therefore, if **A** describes a plane electromagnetic wave then the state of the electromagnetic field can be given as a solution of the Schrödinger equation:

$$\underbrace{\frac{1}{2m}\big(\mathbf{p} - e\mathbf{A}\big)^2 + e\phi\big)}_{H(2.3.1)}\psi = i\partial_t\psi.$$

2.3.1 The Pauli Hamiltonian.

We now derive the non-relativistic quantum dynamical equation of a spin $\frac{1}{2}$ charged particle of charge e in an external vector potential $\mathbf{A}$ and a scalar potential ϕ such that:

$$E \to E - e\phi, \quad \mathbf{p} \to \mathbf{p} - e\mathbf{A}, \quad \psi = \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix}.$$

We can now write equation (2.2.2) in terms of the component spinors

$$\gamma^0(E - e\phi)\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} - \gamma(\mathbf{p} - e\mathbf{A})\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} = m\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix},$$

or

$$(E - e\phi)\begin{pmatrix} \varphi_0 \\ -\chi_0 \end{pmatrix} - (\mathbf{p} - e\mathbf{A})\cdot\begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} = m\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix}.$$

We can express this as two explicit equations given by

$$(E - e\phi)\varphi_0 - \boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})\chi_0 = m\varphi_0, \tag{2.3.4}$$

$$-(E - e\phi)\chi_0 + \boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})\varphi_0 = m\chi_0. \tag{2.3.5}$$

Equation (2.3.5) can be written as

$$\chi_0 = \frac{\varphi_0}{m + E - e\phi}\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A}).$$

We consider the non-relativistic and weak field limits hence $m + E - e\phi \approx 2m$ and

$$\chi_0 \approx \frac{\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})}{2m}\varphi_0.$$

Substituting into (2.3.4), we have

$$E\varphi_0 = \left[\frac{\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})}{2m} + m + e\phi\right]\varphi_0. \tag{2.3.6}$$

Using the identity (2.1.6), we obtain the following relation (Wachter, 2010)

$$(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\sigma}\cdot\mathbf{B}) = \mathbf{A}\cdot\mathbf{B} + i\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B}). \tag{2.3.7}$$

We can express (2.3.6) as $E\varphi_0 = (G + m)\varphi_0$ implying that

$$G\varphi_0 = \underbrace{\left[\frac{\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})}{2m} + e\phi\right]}_{H}\varphi_0. \tag{2.3.8}$$

Applying (2.3.7) to (2.3.8)

$$\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A})\boldsymbol{\sigma}\cdot(\mathbf{p} - e\mathbf{A}) = (\mathbf{p} - e\mathbf{A})^2 + i\boldsymbol{\sigma}\cdot\underbrace{(\mathbf{p} - e\mathbf{A})\times(\mathbf{p} - e\mathbf{A})}_{**}, \tag{2.3.9}$$

the term $**$ when simplified yields

$$(\mathbf{p}\times\mathbf{A}) + (\mathbf{A}\times\mathbf{p}).$$

Taking the natural constant $\hbar = 1$

$$[p_i, A_j] = -i\partial_i A_j,$$

$$\implies (p_i A_j - A_i p_j) + (A_i p_j - p_i A_j) = -i(\partial_i A_j - \partial_j A_i).$$

If we multiply both sides by ε_{ijk} and sum over i and j, we obtain a third component of k hence (Ryder, 1996)

$$\mathbf{p} \times \mathbf{A} + \mathbf{A} \times \mathbf{p} = -i\nabla \times \mathbf{A} = -i\mathbf{B}.$$

Substituting into equation (2.3.9),

$$\boldsymbol{\sigma} \cdot (\mathbf{p} - e\mathbf{A})\boldsymbol{\sigma} \cdot (\mathbf{p} - e\mathbf{A}) = (\mathbf{p} - e\mathbf{A})^2 + i\boldsymbol{\sigma} \cdot (-i\mathbf{B}).$$

We can now write the Hamiltonian in equation (2.3.8) as follows;

$$H = \frac{(\mathbf{p} - e\mathbf{A})^2}{2m} - \frac{e}{2m}\boldsymbol{\sigma} \cdot \mathbf{B} + e\phi.$$

The Schrödinger equation now takes a different form expressed as

$$\left(\frac{(\mathbf{p} - e\mathbf{A})^2}{2m} - \underbrace{g\frac{e}{2m}\mathbf{S}}_{\boldsymbol{\mu}} \cdot \mathbf{B} + e\phi\right)\psi = i\partial_t\psi.$$

This is the **Pauli equation**. In the equation, the vector operator $\mathbf{S} = \frac{\boldsymbol{\sigma}}{2}$ is the spin of the electron, $\boldsymbol{\mu}$ is the magnetic moment. Hence the constant $g = 2$ is the g-value of the electron predicted by Dirac equation.

3. The vertex corrections

In this chapter, we discuss the modification of the vertex by a one-loop Feynman diagram with an internal photon that eventually leads to the calculations of the anomalous electron magnetic moment in the momentum space. What is more, we discuss the first order correction to an electromagnetic vertex.

3.1 The Feyman rules

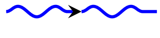
In this section, we introduce the Feynman rules for calculating the scattering matrix $\mathcal{M}$ in the QED theory. The diagrams upon which these rules are based can either be tree or closed diagrams with one or more loops. For our studies, we shall scale down to the specific rules that would lead us to the calculation of g-value from a one-loop correction.

Every Feynman diagram is made up of lines and vertices whereby the straight lines represents a fermion while a wavy line represents a photon with each line propagated from one space-time location to another. The particle propagation from every point are assigned four-momenta which is conserved at the point of interaction(*vertex*). In a nutshell, the rules are summarised as follows:-

- According to Griffiths (2008), notations of the incoming and outgoing four-momentum p and p' as well as the internal momenta q and q' must be done appropriately, and in order to keep track of direction, arrows must be used.
- At every vertex, each particle's momentum must be multiplied by a factor i (Gross, 2008).
- For a photon with a polarisation index of μ, interacting with other particles at a vertex, we use a factor $\pm ie\gamma_\mu$ which is either absorbed or emitted by a fermion of charge $\pm e$ respectively.
- A closed fermion loop is assigned a factor of (-1) and an opposite sign for the same fermion term applies if the fermion lines are exchanged (Wachter, 2010).
- The momentum is always conserved at every vertex and the undetermined 4-momenta are integrated over k if not fixed by energy-momentum conservation with the vertex weight Griffiths (2008), i.e.

$$\int \frac{d^4k}{(2\pi)^4}.$$

- An internally produced momentum-carrying photon (*propagator*) with a momentum k and polarisation indices μ and ν is represented with a factor (Gross, 2008):

$$i\Lambda_P^{\mu\nu}(k) = \frac{ig^{\mu\nu}}{-k^2 - i\epsilon} \Rightarrow$$ .

- For an internal fermion of momentum p, we have a factor of:

$$\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} \Rightarrow \alpha$$  ρ.

- For fermions, we assemble the incoming and outgoing fermion spinors, vertex operators, along each fermion line in order to make a matrix element.

3.2 Virtual particles

In the study of the sub-atomic particles, we encounter the interactions of the force carrying particles at the microscopic level. For example, excitation of the electromagnetic fields between particles mediates the resultant Coulomb force between them. In this case, the distance between the interacting(repulsive/ attractive) particles is very difficult to visualize and therefore the process can only be possible as a result of exchange of some unseen photons.

There is therefore need to understand where the photons originates, and where they go to afterwards.

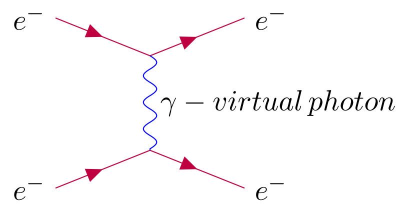


Figure 3.1: *A virtual particle in an elastic electron scattering*

The mediating photons are emitted by one electron and absorbed by another. It is not easy to see which electron emits or which ones absorbs but the effects of the processes involved can tell. The force carrying particles (messenger particles) are the *virtual particles*. They exhibits properties different from those of the real particles.

These unusual properties of virtual particles can be illustrated by considering a single virtual photon involved in an elastic scattering (Gross, 2008). If we assume that the nucleus is more massive, it appears to be approximately stationary. If the incoming electron has a momentum $\mathbf{p}$, and the outgoing electron has a momentum $\mathbf{p}'$. When the electron of momentum $\mathbf{p}$ absorbs the virtual photon, it experiences a change in momentum given by $\delta\mathbf{p} = \mathbf{p}' - \mathbf{p}$ and unchanged energy i.e. $E' = E$.

According to Barr (2014), the final energy and momentum of the photon after the process is given by: $E_\gamma = 0$ and $\mathbf{p}_\gamma = \mathbf{p}' - \mathbf{p} = \delta\mathbf{p}$ respectively. From this, we observe that $E_\gamma^2 \neq p_\gamma^2$ (off-shell). What is more, they exhibit an energy-momentum invariance which is not equal to the square of the mass

$$p.p = E^2 - \mathbf{p}\cdot\mathbf{p} \neq m^2.$$

They are therefore said to be "off mass shell".

3.3 Electron's magnetic moment and the Lande's g-factor

If we consider a particle scattering from another heavier particle, the strength of the electromagnetic constant can be measured by considering a low energy elastic scattering process called Mott scattering Nagashima and Nambu (2011) and in this case, the matrix element of the process can be written as follows

$$i\mathcal{M} = \overline{u}(p')ie\Gamma_\nu u(p)i\frac{g^{\mu\nu}}{q^2}\overline{u}(k')ie\gamma_\mu u(k), \tag{3.3.1}$$

where $\overline{u}(p')$ and $u(k)$ are the solutions to Dirac equations for the scattering electron and the heavier particle respectively and Γ_ν is a representation of all the vertex corrections. For the case of this study, we consider the process that provides the leading order corrections to the vertex.

In the diagram, p and p' are the momenta of the incoming and scattered electrons respectively and $q = p' - p$ is the momentum of the virtual photon discussed in section 3.2 above. The goal now is to compute the vertex corrections Γ_ν

In order to attain the goal, we need to carry out an explicit loop computation for Γ_ν. The form that it takes can be evaluated using symmetry properties e.g. Lorentz transformation. In this case, $\Gamma_\mu = \gamma_\mu$ for the lowest order.

In general, $\Gamma^\mu = \Gamma^\mu(\gamma^\mu, p^\mu, p'^\mu)$. Hence we can express it as a linear combination of other scalar functions $A(p,p')$, $B(p,p')$ and $C(p,p')$ (Tripathy, 2013) i.e.

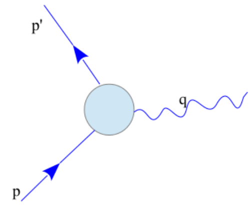


Figure 3.2: *General vertex*

$$\Gamma^\mu = \gamma^\mu A(p,p') + (p^\mu + p'^\mu)B(p,p') + (p^\mu - p'^\mu)C(p,p'). \tag{3.3.2}$$

From here, we can get the exact corrections by evaluating the values of scalar functions A, B and C to all orders. Next, we apply a Ward-Takahashi identity for a three-point vertex function (since it holds even if the photon is off-shell) (Nagashima and Nambu, 2011).

$$q_\mu \Gamma^\mu = 0.$$

Proof. To show that $C(p,p') = 0$, we apply Ward -Takahashi identity and Lorentz conservation laws to equation (3.3.2) and hence we have:

$$q_\mu \Gamma^\mu = \not{q}A + q \cdot (p + p')B + q \cdot (p - p')C = 0.$$

Since $q = p' - p$ and $p'^2 = p^2 = m^2$ (on-shell electrons), we note that

$$q \cdot (p + p') = 0.$$

From Dirac equation, $(\not{p} - m)u(p) = 0$ hence,

$$\overline{u}(p')\not{q}u(p) = \overline{u}(p')(\not{p}' - \not{p})u(p) = \overline{u}(p')(m - m)u(p) = 0,$$

For the last term,

$$q \cdot (p - p') = -q^2$$

$\implies p'^2 = p^2 + 2pq + q^2$, hence $q^2 + 2pq = 0$, and from here, we see that $q^2 \neq 0$.

Therefore, the term $q \cdot (p - p') \neq 0$, implying that $C = 0$ for equation (3.3.2) to be consistent. □

Substituting $C(p,p') = 0$ into equation (3.3.2), we have the final correction function expressed as

$$\Gamma^\mu = \gamma^\mu A(p,p') + (p^\mu + p'^\mu)B(p,p'). \tag{3.3.3}$$

We now apply Gordon identity to rewrite $p^\mu$and p'^μ dependence in terms of $\sigma^{\mu\nu}$ since it holds for on-shell spinors and this results to

$$\overline{u}(p')\gamma^\mu u(p) = \overline{u}(p')\left[\frac{p^\mu + p'^\mu}{2m} + i\frac{\sigma^{\mu\nu}}{2m}q_\nu\right]u(p). \tag{3.3.4}$$

In the above equation, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ is the generator for the Lorentz transformation. Since the p' and p dependence is only brought about by q^2, we can express equation (3.3.4) as

$$\Gamma^{\mu} = \gamma^{\mu} F_1(q^2) + i\frac{\sigma^{\mu\nu}}{2m} q_{\nu} F_2(q^2), \tag{3.3.5}$$

where F_1 and F_2 are some functions of q^2 only in order to maintain Lorentz covariance. They are known as *form factors* and F_1 shows the effect of all corrections to the charge (charge re-normalization) while F_2 illustrates the effect of corrections to the magnetic moment. Now, we have to find the form factors by computing the loop correction.

In the non-relativistic limit, where the momentum transferred from the heavy particle $q \to 0$ (since the magnetic field is weak), $F_1(q^2)$ will not receive any corrections whereas $F_2(q^2)$ will receive non-trivial corrections (Gross, 2008). If we consider an electron scattering from an external electromagnetic field, we assume a static vector potential so that we have only an external magnetic field. The interaction Hamiltonian can therefore be expressed as

$$H_{int} = \int d^3x e A_{\mu}^{cl} j^{\mu},$$

where $j^{\mu} = \overline{\psi}(x)\gamma^{\mu}\psi(x)$. We take a classical potential A_{μ}^{cl} since the external field is not quantised. The Feynman amplitude for the leading order correction can now be computed as

$$i\mathcal{M}(2\pi)\delta(p_0' - p_0) = -ie\overline{u}(p')\gamma^{\mu}u(p)\widetilde{A}_{\mu}^{cl}(p' - p),$$

where $\widetilde{A}_{\mu}^{cl}$ is the Fourier transform of A_{μ}^{cl}. For our case, γ^{μ} is represented by Γ^{μ} and the potential $\widetilde{A}_{\mu}^{cl}(x) = (0, \mathbf{A})$ hence the amplitude is given by

$$i\mathcal{M} = \overline{u}(p')ie\Gamma^{\mu}u(p)\widetilde{A}_{\mu}^{cl}(\mathbf{q}) = \overline{u}(p')ie\left[\gamma^i F_1(q^2) + i\frac{\sigma^{i\nu}}{2m}q_{\nu}F_2(q^2)\right]u(p)\widetilde{A}_{cl}^{i}(\mathbf{q}),$$

where $\widetilde{A}_{cl}^{i}(\mathbf{q}) = -\widetilde{A}_{cl}^{\mu}(x)$. In the non-relativistic limit, where q is very small,

$$i\mathcal{M} = \overline{u}(p')ie\left[\gamma^i F_1(0) + i\frac{\sigma^{i\nu}}{2m}q_{\nu}F_2(0)\right]u(p)\widetilde{A}_{cl}^{i}(\mathbf{q}). \tag{3.3.6}$$

We now take the explicit solution of Dirac equation generally expressed as equation (2.2.6), where the two component spinors discussed above are now represented by $\varphi(0)$. We will now work in a representation where the γ-matrices are given by equation (2.1.8) and assume that the momenta are all small compared to the mass of the electron. Therefore, the first term in equation (3.3.6) can be expressed as

$$\overline{u}(p')\gamma^i u(p) = u^{\dagger}(p')\gamma^0\gamma^i u(p) = u^{\dagger}(p')\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}u(p).$$

This implies that

$$\overline{u}(p')\gamma^i u(p) = \begin{pmatrix} \varphi'^{\dagger}(0)\sqrt{\frac{E'+m}{2m}} & \varphi'^{\dagger}(0)\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\sqrt{2m(E'+m)}} \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}\begin{pmatrix} \sqrt{\frac{E+m}{2m}}\varphi(0) \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\sqrt{2m(E+m)}}\varphi(0) \end{pmatrix}$$

after multiplying out and simplifying using the assumption that at non-relativistic limit $E' = E$ we get

$$\overline{u}(p')\gamma^i u(p) = \frac{1}{2}\varphi'^{\dagger}(0)\left[\boldsymbol{\sigma}\cdot\mathbf{p}'\sigma^i + \sigma^i\boldsymbol{\sigma}\cdot\mathbf{p}\right]\varphi(0).$$

We now apply the commutator-anti commutator relation identity (2.1.6) to simplify the equation further, the equation can be rewritten as

$$\overline{u}(p')\gamma^i u(p) = \frac{1}{2}\varphi'^{\dagger}(0)\left[\sigma^j\sigma^i p'^j + \sigma^i\sigma^j p^j\right]\varphi(0),$$

$$\overline{u}(p')\gamma^i u(p) = \frac{1}{2}\varphi'^{\dagger}(0)\left[(p'+p)^i + i\varepsilon^{jik}\sigma^k(p'-p)^j\right]\varphi(0).$$

On further simplification, we only consider the terms that are linear in q_j and hence we obtain the **first term** to be

$$\overline{u}(p')\gamma^i u(p) = \varphi'^{\dagger}(0)\left[-\frac{1}{2}i\varepsilon^{ijk}q^j\sigma^k\right]\varphi(0) + \cdots .$$

For the second term, we need to evaluate

$$\overline{u}(p')\sigma^{i\nu}q_\nu u(p) = \overline{u}(p')\sigma^{ij}q_j u(p) + \cdots . \tag{3.3.7}$$

In the standard Weyl representation, $\sigma^{ij} = \frac{i}{2}[\gamma^i,\gamma^j] = \varepsilon^{ijk}\begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix}$. We now multiply both sides of the equation (3.3.7) by $\frac{i}{2m}$ and take only the terms that are linear in q_j

$$\frac{i}{2m}\overline{u}(p')\sigma^{ij}q_j u(p) + \cdots = \frac{i}{2m}u^{\dagger}(p')\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}\begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix}u(p)\varepsilon^{ijk}(-q^j) + \cdots .$$

On further simplification using the procedures above, we obtain the **second term** as

$$\frac{i}{2m}\overline{u}(p')\sigma^{i\nu}q_\nu u(p) = \varphi'^{\dagger}(0)\left[-\frac{i}{2m}\sigma^k\varepsilon^{ijk}q^j\right]\varphi(0) + \cdots .$$

We have now evaluated the terms in the Feynman amplitude and therefore we substitute them back into the original equation (3.3.6) to get

$$i\mathcal{M} = -ie(2m)\varphi'^{\dagger}(0)\left(-\frac{1}{2m}\sigma^k\big(F_1(0)+F_2(0)\big)\underbrace{i\varepsilon^{ijk}q^j\tilde{A}^i_{cl}(\mathbf{q})}_{*}\right)\varphi(0). \tag{3.3.8}$$

Since $\mathbf{B}(x) = \nabla\times\mathbf{A}(x)$. We notice that the term marked * is the Fourier transform of $\mathbf{B}^k(x)$ which we can express as $i\varepsilon^{ijk}q^j\tilde{A}^i_{cl}(\mathbf{q}) = \tilde{B}^k(\mathbf{q})$ and the amplitude is now expressed as

$$i\mathcal{M} = -ie(2m)\varphi'^{\dagger}(0)\left(-\frac{1}{2m}\sigma^k\big(F_1(0)+F_2(0)\big)\right)\tilde{B}^k(\mathbf{q})\varphi(0). \tag{3.3.9}$$

According to Potamianos (2007), the Born approximation of the scattering amplitude is given by the Fourier transform of the scattering potential, hence we can write the Born approximation of equation (3.3.9) as

$$U = -\langle\boldsymbol{\mu}\rangle\cdot\mathbf{B}.$$

This implies that

$$\langle\boldsymbol{\mu}\rangle = \frac{e}{2m}2\big(F_1(0)+F_2(0)\big)\varphi'^{\dagger}(0)\frac{\boldsymbol{\sigma}}{2}\varphi(0).$$

We now introduce a constant g (Landes' g-factor) and the spin operator $\mathbf{S} = \varphi'^{\dagger}(0)\frac{\boldsymbol{\sigma}}{2}\varphi(0)$ to obtain

$$\langle \boldsymbol{\mu} \rangle = g\left(\frac{e}{2m}\right)\mathbf{S}. \tag{3.3.10}$$

From this equation (3.3.10), we can deduce the value of g as

$$g = 2\big[F_1(0) + F_2(0)\big].$$

In a 1-loop correction, $F_1(0) = 1$ since it doesn't receive any corrections, implying that:

$$g = 2 + 2F_2(0). \tag{3.3.11}$$

The first term, 2 represents the value of g predicted by the Dirac theory (without corrections) while the second term $2F_2(0)$ is the anomaly resulting from the corrections explained in quantum theory. The main task is now to determine $F_2(0)$ and hence the anomalous magnetic moment.

3.4 Calculation of the anomalous magnetic moment $(2F_2(0))$

As stated above, we will calculate $2F_2(0)$ from a 1-loop correction to the vertex vector by the help of the diagram shown

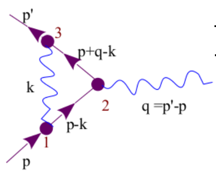


Figure 3.3: *Feynman diagram for a 1-loop correction*

Using the Feynman identity for the amplitude and external electron states, we can express the amplitude as an integral over the product of individual corrections at the vertices 1, 2 and 3 and then integrate over k (Schwartz, 2013; Gross, 2008). This is given by

$$i\mathcal{M}_\Gamma = ie\overline{u}(p')\Gamma^\mu u(p)$$

$$i\mathcal{M}_\Gamma = \overline{u}(p')\frac{(-ie)^3}{(2\pi)^4}\int d^4k\left[\frac{(-1)\gamma^\alpha(g_{\nu\alpha})}{\beta^2 - k^2 - i\epsilon}\right]\left[\frac{\gamma^\mu(m + \not{p} - \not{k})}{m^2 - (p-k)^2 - i\epsilon}\right]\left[\frac{\gamma^\nu(m + \not{p}' - \not{k})}{m^2 - (p'-k)^2 - i\epsilon}\right]u(p)$$

where β is the arbitrary mass of the virtual photon we introduce to prevent any contributions from very soft photons which could otherwise lead to infra-red divergences. The photon is emitted at 1 and reabsorbed at 3 figure 3.3. To simplify the equation, we now use the Dirac equation as in (Gross, 2008)

$$\not{p}u(p) = mu(p) \text{ and } u(p)\not{p}' = mu(p).$$

We therefore have

$$i\mathcal{M}^\mu = \overline{u}(p')\frac{ie^2}{(2\pi)^4}\int d^4k\frac{\gamma_\nu(m + \not{p} - \not{k})\gamma^\mu(m + \not{p}' - \not{k})\gamma^\nu}{[m^2 - (p-k)^2 - i\epsilon][m^2 - (p'-k)^2 - i\epsilon][\beta^2 - k^2 - i\epsilon]}u(p). \tag{3.4.1}$$

According to Gross (2008), a 1-loop integral of the above form can be evaluated by:

- Combining the denominators into a single denominator and then reducing to the standard form by shifting the loop momenta.

- The standard integral can then be evaluated by use of an identity, generally expressed as

$$\frac{1}{ABC} = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{2}{\left[x_1 A + Bx_2 + C(1 - x_1 - x_2)\right]^3}. \tag{3.4.2}$$

Now, since both the incoming and the outgoing electrons are on-shell, $p'^2 = p^2 = m^2$ hence,

$$A = m^2 - (p' - k)^2 - i\epsilon = 2p'.k - k^2 - i\epsilon$$

$$B = m^2 - (p - k)^2 - i\epsilon = 2p.k - k^2 - i\epsilon$$

$$C = \beta^2 - k^2 - i\epsilon.$$

These are the Feynman's parameters. If D is the denominator and N^μ is the numerator, we write

$$D = -k^2 + 2(x_2 p + x_1 p').k + \beta^2(1 - x_1 - x_2) - i\epsilon.$$

We will now complete the square in D by shifting k such that $k = k' + x_1 p' + x_2 p$ while at N^μ we shift $k \to k'$ and then substitute back Gross (2008), hence we have

$$D = (x_1^2 + x_2^2)m^2 + 2x_1 x_2 p.p' + \beta^2(1 - x_1 - x_2) - k'^2 - i\epsilon$$

and

$$N^\mu = \gamma^\nu\left[m + \not{p}'(1 - x_1) - x_2\not{p} - \not{k}'\right]\gamma^\mu\left[m + \not{p}(1 - x_2) - x_1\not{p}' - \not{k}'\right]\gamma_\nu.$$

Since D is an even function of k', we now drop the terms in N^μ that are linear in k' as they will integrate to zero. We therefore have:

$$N^\mu = \gamma^\nu\left[m + \not{p}'(1 - x_1) - x_2\not{p}\right]\gamma^\mu\left[m + \not{p}(1 - x_2) - x_1\not{p}'\right]\gamma_\nu + \gamma^\nu\not{k}'\gamma^\mu\not{k}'\gamma_\nu.$$

We will now use the identities of the γ matrices listed below (Kumericki, 2001)

- $\not{p}\not{p} = p.p.I$.
- $\gamma^\mu\gamma_\mu = 4I$.
- $\gamma^\mu\not{p}\gamma_\mu = -2\not{p}$.
- $\gamma^\mu\not{p}\not{q}\gamma_\mu = 4p.q$.
- $\gamma^\mu\not{p}\not{q}\not{r}\gamma_\mu = -2\not{r}\not{q}p$.
- $\not{q}\gamma^\mu\not{q} = 2q^\mu\not{q} - \gamma^\mu q^2$.

We can now apply the above identities to reduce the numerator to

$$N^\mu = -2m^2\gamma^\mu + 4m\left[p'^\mu(1 - x_1) - x_2 p^\mu + p^\mu(1 - x_2) - x_1 p'^\mu\right]$$

$$-2\left[\not{p}(1 - x_2) - x_1\not{p}')\gamma^\mu(\not{p}'(1 - x_1) - x_2\not{p}\right] - 2\not{k}'\gamma^\mu\not{k}'.$$

We now use the identities $\not{p} = \not{p}' - \not{q}$ and $\not{q}\gamma^\mu\not{q} = 2q^\mu\not{q} - \gamma^\mu q^2$ together with algebraic simplifications. According to Gross (2008), the $x_2 - x_1$ terms integrates to zero as $q \to 0$ and the result is given as

$$N^\mu = \gamma^\mu\big(-2m^2 - 2m^2(1-x_1-x_2)^2 - 2(1-x_2)(1-x_1)q^2 + 2k'^2\big) - 4k'^\mu \not{k}' + 4m(1-x_1-x_2)(p'+p)^\mu$$
$$-2m(1-x_1-x_2)(1-\frac{1}{2}(x_1+x_2))[\gamma^\mu, \not{q}].$$

For further simplification, we use the identities $\frac{i}{2}[\gamma^\mu, \not{q}] = \sigma^{\mu\nu}q_\nu$ and $p'^\mu + p^\mu = 2m\gamma^\mu - i\sigma^{\mu\nu}q_\nu$, hence

$$N^\mu = \gamma^\mu\Big(-2m^2(1-x_1-x_2)^2 + \big(1-4(1-x_1-x_2)\big) - 2q^2(1-x_1)(1-x_2) + 2k'^2\Big) - i2m\sigma^{\mu\nu}q_\nu(1-x_1-x_2)(x_1+x_2).$$

Substituting D and N^μ into the equation (3.4.1), we have

$$i\mathcal{M}^\mu = \overline{u}(p')2ie^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int \frac{d^4k'}{(2\pi)^4} \frac{N^\mu}{\big[D(k')\big]^3} u(p). \tag{3.4.3}$$

Comparing this form of amplitude to that of equation (3.3.5) and taking into account the form of N^μ, we notice that $i\mathcal{M}^\mu$ can be conveniently written in terms of $F_1(q^2)$ and $F_2(q^2)$ terms. However, we have already seen for this case that $F_1(q^2 \to 0) = 1$ and we need now to compute $F_2(q^2 \to 0)$. In order to compute this term, we consider the terms that are a coefficient of $i\sigma^{\mu\nu}q_\nu$ at the numerator (N^μ). This is expressed as

$$F_2(q^2) = -2ie^2 \frac{1}{(2\pi)^4} \int d^4k' \int_0^1 dx_1 \int_0^{1-x_1} \frac{4m^2(x_1+x_2)(1-x_1-x_2)}{\big[D(k')\big]^3} dx_2$$

where at $q^2 = 0$,

$$D = (x_1+x_2)^2m^2 + \beta^2(1-x_1-x_2) - k'^2 - i\epsilon$$

so that

$$F_2(0) = -2ie^2 \frac{1}{(2\pi)^4} \int d^4k' \int_0^1 dx_1 \int_0^{1-x_1} \frac{4m^2(x_1+x_2)(1-x_1-x_2)}{[(x_1+x_2)^2m^2 + \beta^2(1-x_1-x_2) - k'^2 - i\epsilon]^3} dx_2. \tag{3.4.4}$$

For further simplification, we use the identity (Schwartz, 2013)

$$\frac{i}{(2\pi)^4} \int d^4k' \frac{1}{[-k'^2 + \triangle - i\epsilon]^3} = \frac{-1}{32\pi^2\triangle^2}$$

where, for our case,

$$\triangle = (x_1+x_2)^2m^2 + \beta^2(1-x_1-x_2).$$

According to Gross (2008), the singularity at $x_1 + x_2 = 0$ is only a point in a two-dimensional space hence integrable. If the virtual photon's mass $\beta \to 0$ and the coupling constant $\alpha = \dfrac{e^2}{4\pi}$, we can change the variables as follows

$$\kappa = x_1 + x_2 \quad \text{and} \quad \Omega = \frac{x_1 - x_2}{2\kappa}.$$

The volume element then transforms to

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 = \int_0^1 \kappa d\kappa \int_{-\frac{1}{2}}^{\frac{1}{2}} d\Omega.$$

Therefore we can write equation (3.4.4) as

$$F_2(0) = \frac{\alpha}{\pi}\int_0^1 d\kappa \int_{-\frac{1}{2}}^{\frac{1}{2}} (1-\kappa) d\Omega = \frac{\alpha}{2\pi}, \tag{3.4.5}$$

$$2F_2(0) = \frac{\alpha}{\pi}.$$

This is **the anomalous electron magnetic moment**. In this case, the coupling constant is the fine structure constant ($\alpha \approx \frac{1}{137}$). By combining equation (3.3.11) and equation (3.4.5), we obtain the gyromagnetic ratio for the electron from the 1-loop correction, expressed as

$$g = 2 + \frac{\alpha}{\pi} \approx 2.00232. \tag{3.4.6}$$

The corrections of higher order can be calculated in the same way by use of more complex Feynman diagrams. However, the 1-loop correction we have used was a great milestone historically in the quantum electrodynamic field theory, it actually convinced people that loops have physical effects.

3.5 Discussion of higher order corrections

Higher order corrections includes vertex corrections (calculated above), photon propagator corrections, fermion propagator corrections as well as corrections due to emission of very low energy photons. For diagrams with loops, two types of divergences of the integrals generally arises when calculating these higher corrections using Feynman diagrams, these are **ultra-violet divergence** (i.e. the integral diverges like $\sim \log k^2$ as $k \to \infty$) and **infra-red divergence** (i.e. the integral diverges as $k \to 0$) (Nagashima and Nambu, 2011). These divergences we encounter can be explained from physical point of view to be as a result of high energy limits of the momentum integration and infinitely many emitted soft photons.

Ultraviolet divergence is concentrated around the F_1 term and therefore it entirely affects the charge of the electron and not its magnetic moment. In order to deal with this divergence, we use the charge *re-normalization* process which enables us to effectively calculate the effects of low energy physics irrespective of its correction at higher energies (Waldram, Accessed May 2014). In this procedure, we subtract the value $F_1(0)$ from the first term, which ensures that the remainder is zero at $q^2 = 0$, and hence the charge remains unaffected (Gross, 2008). Mathematically, at higher orders, we express equation (3.3.5) as

$$\Gamma^\mu = \big(F_1(q^2) - F_1(0)\big)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m} q_\nu F_2(q^2) + F_1(0)\gamma^\mu$$

where the infinite constant $F_1(0)$ becomes absorbed by the multiplicative re-normalization factor

$$Z_1 = \frac{1}{F_1(0) + 1}.$$

We also note that there are two possible corrections that can take place around a single charge and they include (Gross, 2008):

- *Proper vertex corrections (one particle irreducible)*-These corrections cannot be divided into two by cutting a propagator or electron line.
- *Improper corrections*-These corrections can be divided into two by a propagator or electron line.

For the calculations in sections 3.3 and 3.4 above, we have considered only the proper corrections but for higher order correction calculations, the sum of all Feynman diagrams contributing to vertex correction must be considered and expressed in terms of proper vertex.

On the other hand, **infra-red divergence** cancel out with the emissions of very low energy photons, it vanishes to order α if we include virtual photon mass β contribution to the vertex (Nagashima and Nambu, 2011).

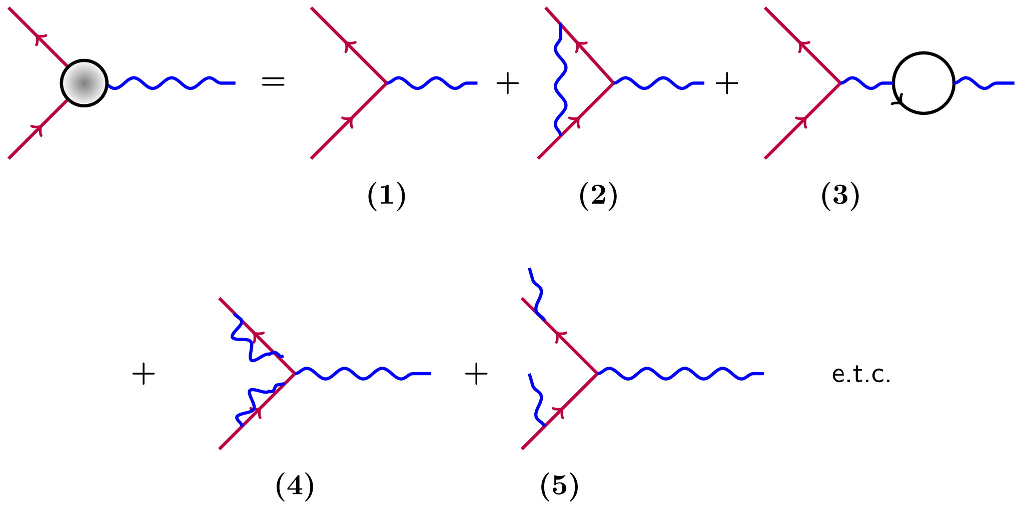


Figure 3.4: Diagrams of higher order correction to the vertex in QED.

From the figures above:

- (1)- represents a bare vertex without any loop hence no correction.
- (2)- represents the vertex correction we have used in our calculations.
- (3)- represents the photon propagator correction.
- (4)- represents the fermion propagator correction.
- (5)- represents the correction as a result of emission of very low energy photons.

4. Applications

In this chapter, we now discuss some of the applications of the electron's magnetic moment and spin in the modern technology.

4.1 Spintronics and hard drives

Spintronics is a new field of nanotechnology in which the spin of an electron in a magnetic field is used to increase the storage capacity in mass-storage devices such as computer hard discs (Spi, 2014).

In order to combat the increasing demand for data storage, catalysed by the rapidly growing technology, Albert Fert and Peter Grunberg invented the Giant Magneto-resistance (GMR) in 1988. Building on this discovery, Stuart Parkins, an English experimental physicist sought to come up with a technologically high capacity magnetic disc drive, which could occupy a lesser physical space while increasing the storage capacity (Spi, 2014).

The scientific principle behind discovery was that the flow of electric current could be controlled by manipulating the spin of the electrons, which could only be possible when the electron was in a magnetic field where all the spins are oriented in the same direction as the magnetisation i.e. the magnetic moment and the spin directions are not conserved even if the charge conservation holds.

So, by creating the materials in atomic scale, with the distinct magnetic and non-magnetic regions, we can effectively control the flow of current of spin polarised electrons and hence devise materials whose resistance changes in the presence of magnetic field. Moreover, this newly invented hard disc is able to detect smaller magnetic fields, which enhances tremendously the storage of data capacity. This huge expansion in storage capacity has been a determining factor in big data revolution as it comes with other opportunities of gathering and analysing data.

4.2 Electron spin resonance (ESR)-Conduction electron detection

ESR is a technique used in sciences for studying the materials with unpaired electrons. It is based on the fact that an electron is a charged particle that spins in its axis when placed in a magnetic field thus acting as a micro-magnet with both the north and the south poles (Farach and Poole, 2014).

Since an electron has a spin $s = \frac{1}{2}$, the magnetic moment of the electron is aligned either parallel (corresponding to spin $s = \frac{1}{2}$) or in anti-parallel (corresponding to $s = -\frac{1}{2}$) directions relative to the field, in the presence of an external magnetic field $\mathbf{B}$. Since each of these quantum states has a distinct energy, an application of a small external magnetic field at a microwave frequency enables the unpaired electrons to move between states by either absorbing or emitting a photon at a frequency equal to the resonance frequency of the incumbent field (Weil and Bolton, 2007). The resonance absorption of energy will take place if:

$$\triangle E = E_{\frac{1}{2}} + E_{-\frac{1}{2}} = g\mu_B \mathbf{B_0}(1 - \sigma)$$

where μ_B is the Bohr's magneton and σ is the effect of the induced local field.

In detecting the conduction electrons, an ESR spectrometer and a sample is placed in a region of strong magnetic field strength, after which the degree of circular polarisation of the luminescence is measured (Farach and Poole, 2014). As a result, we obtain various line spectra which when studied, we can obtain information about the spin exchange between the identical and non-identical molecules.

5. Conclusions

We have presented a theoretical method of determining the anomaly in the electron magnetic moment predicted in QED theory. To achieve this, we have made use of Feynman rules and diagrams introduced in 1948 by Richard Feyman 3.1. This has prompted us to carry out algebraic manipulations in order to simplify the integrals involved in calculating the form factors from the loop correction to the vertex. The results obtained here are considered complete because the value of $g \approx 2.00232$ (5 d.p) is in a complete agreement with QED theory which is the most accurate physical theory in particle physics 1.1, 3.4.

The existence of electron's magnetic moment and intrinsic spin has been precisely discussed in details in section 1.1. The electron being one of the leptons in the standard model (Best, 2014), experiences a force (torque) while in an electromagnetic field via its magnetic moment and hence the spin which are related through the gyromagnetic ratio, g 1. In chapter 2, we have discussed the Dirac equation which we have found to be form-invariant under Lorentz transformation 2.1. Furthermore, the canonical form of Dirac equation exhibits an explicit solution with both positive and negative values for the energy eigenvalues, with the component spinors 2.2. The positive energy solutions have been vital in determining the form factors F_1 and F_2. The Pauli matrices and gamma matrices have been introduced in section 2.1 and effectively used in section 3.4 to manipulate the loop integrals. What is more, a non-relativistic consideration of the electron can be achieved through the coupling of the canonical form of Dirac equation with an external magnetic field field, resulting to Pauli equation 2.3.

Some of the applications of electron's magnetic moment and spin in technology have been discussed in chapter 4. We have seen that the storage capacity of the computer hard-drives have increased to a greater extent through the use of spintronics. In addition, ESR, an emerging technology which uses the principles of spin and magnetic moment of an electron to detect the conductivity of materials has seen a great advancement in electronic industry and the field of science as a whole.

Loops have physical implications in QED as revealed by the result in section 3.4. However much can still be done to improve the understanding of these effects. There are no clear physical applications of the fact that $g \neq 2$ in technology and this can still be probed. In addition, we have only used the correction loops to determine the g-value for an electron which is just but one of the leptons and therefore, this work can be easily extended to study other leptons i.e. muons and taus and even more composite particles such as neutrons and protons.

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