Construction of Magnetic Resonance Images

by

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Abstract

The use of radio frequency (RF) pulses and a strong static magnetic field to provide remarkable high quality and detailed images of internal organs and tissues has been the goal in magnetic resonance imaging (MRI).

We discuss the mathematics involved in the reconstruction of simple $T_1$ and $T_2$ proton MRI images, and a discussion on $T_1$ and $T_2$ weighted images. An understanding of the quantum property “spin”, observed in nuclei of all isotopes except those with even number of both protons and neutrons is demonstrated. The spin causes the nuclei to create their own magnetic dipoles.

The axes of the nuclear dipoles rotate around the N-S axis of the static external magnetic field at a rate depending on the isotope. The manipulation of the magnetisation by an RF pulse applied along the orthogonal plane and coinciding with the rotational rate of precessing dipole is the key to MR imaging.
Dedication

This work is dedicated to God, who has led me this far and to my loving parents whose constant support I cherish.
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Chapter 1

INTRODUCTION

Background

Historically, different abbreviations have been used to describe the process of recording the induced absorption and emission of energy from nuclei placed in a magnetic field. Around 1940, it was referred to as nuclear induction, but was changed in about 1950 to nuclear paramagnetic resonance. At about the late 1950s, the term nuclear magnetic resonance (NMR) became the preferred name for this process [1].

At the time imaging methods using NMR signal was first developed, the name NMR imaging was used. However, this was dropped as a result of negative reactions from patients, who view the term “nuclear” as posing some danger. This happened within the 1980s and magnetic resonance imaging (MRI) became the preferred name for this new technique.

Currently, NMR is used when describing the physical phenomenon itself or when referring to measurements of nuclear induction signals in laboratories. When dealing with people or animals in imaging, MRI is more frequently used.

MRI is used principally in a clinical setting to produce high quality images of the inner part of the human body. This technique relies upon the principle of nuclear magnetic resonance (NMR). It is used to obtain microscopic chemical and physical information about molecules.

The nucleus of an atom is made up of protons, which are positively charged, and neutrons, which are neutral. The nucleus may possess a magnetic property as a result of a quantum mechanical effect known as spin. This depends on the number of protons and neutrons in the nucleus. Nuclei with an even number of both protons and neutrons, will have no net spin. If this is not the case, there will be a net spin associated with the nuclei. Their spin and charge distribution generate a magnetic field in the nucleus. The manipulation of the magnetic properties of the nucleus within an external static magnetic field, by some applied RF pulse is a major concern in MRI.

Resonance is the transfer of energy from a system to another as a result of coincidence in their
characteristic frequencies. In NMR, this coincidence occurs when the frequency of an applied radio wave coincides with the precession frequency of the nuclear dipole.

The detection and acquisition of signals from NMR have been extensively studied since 1939 and have been used as an analytic tool in both chemistry and biochemistry. At this time, Isidor Isaac Rabi and his research group used this technique in the study of atomic and molecular properties. They used their knowledge of physics, particularly that atoms are made up of electrons, protons and neutrons, and that they spin about their axes to conduct research [2]. The resultant spin angular momentum has a magnitude and direction; The combination of spin and charge generate a magnetic field, analogous to that of a tiny bar magnet with North and South poles. When subjected to an external magnetic field, the nuclear magnetic field generates a moment which tends to the nuclear magnetic dipole align in the direction of the external field.

Rabi’s experiment involved passing a beam of lithium chloride molecules through a vacuum chamber and manipulating the beam with different magnetic fields. He investigated the behaviour of the magnetic moment of the nucleus by studying how the magnetic field affected the path of the molecules. He then predicted that the nuclear magnetic dipole could be tipped, relative to the external magnetic field, to a selected angle. This can be thought of as the excitation of the spin states. When the source of excitation is removed, it tries to return to its initial state (relaxation) and in the process, emits photons which are observed as NMR signals.

NMR is a physical phenomenon base upon the magnetic properties of an atoms nucleus. The most commonly used nuclei are those of hydrogen and carbon, but for clinical purposes, hydrogen nuclei is preferred and the technique becomes MRI. This is because the major constituents of the human body are fat and water. These consist primarily of hydrogen atoms and make up about 63 percent of the human body. For these reasons MRI primarily images signals from the hydrogen nuclei. Some striking examples of magnetic resonance imaging are:

1. Long slices, particularly for the spine.
2. High intensity 3-D images of vessels, as in blood flow.
3. Differences in chemical constituents between tissues showing up as changes in a gray scale image.

Fast imaging techniques that curb movement artifact so as to enhance the quality of images have been developed. The signals collected using this technique have a very strong correlation with the physiological properties of the tissue [3].

Most MRI systems are made up of a radio frequency (RF) receiver for the detection of signals, an RF transmitter for the transmission of RF pulse, small field of view (FOV) receiving coils for specific parts of the anatomy, a large bore magnet (resistive, permanent, or superconducting, with the superconducting showing many advantages), and a computer and array processor with fast Fourier transform (FFT) software (used to construct images from signal matrices).

MRI has supplanted many computer tomography and projection radiographic methods because of
its high contrast sensitivity to soft tissue differences. It is harmless, since it doesn’t use ionizing radiations. This is a major advantage. Since its invention, MRI has improved tremendously in image quality, signal acquisition methods and equipment design. Its problems, however, include equipment and siting costs, the complexity of the equipment, patient claustrophobia and relatively long imaging time.

1.1 MRI safety

The detection of metallic materials within patients before MR imaging is very important because their movement may result in serious injury. The patients clinical history is provided to ensure that there has not been any implant of metallic foreign body in the patient. Metal detectors are also used for further verification of the presence of materials in the body [1]. Also, there have been several considerations necessitating the exclusion of patients with cardiac pacemaker from MR imaging. This is strictly because, a pacemaker contains several magnetically and electrically sensitive electronic components that may potentially be rendered inoperative when placed in the scanner. This may lead to death, and at least two deaths have been reported during the MR imaging of patients with pacemakers [4] There is also the issue of dental materials and devices. Many of such materials are ferromagnetic and create local artifact during MR imaging. In order that this artifact is minimised, patients are advised to remove such bridgework before imaging.

There is also the issue of patient’s claustrophobia. Various measure have been proposed to reduce the anxiety in patients, but till date there has been no perfect way of doing this. These maneuvers include providing patients education, blindfolding, piping in music, allowing a family member to accompany the patient, maintaining adequate verbal contact and applying various psychological desensitisation techniques. These method have been used for various patients, but Valium is used for the very difficult ones [5].
Chapter 2

MAGNETIC PROPERTIES AND MRI

2.1 Magnetic strength and types

The main component of an MRI system is the magnet, used to create the static external field, and three types are used: permanent, resistive, and superconducting.

The unit of magnetic flux density is the Gauss $G$, or Tesla, $T$ (1000 Gauss=1 Tesla). The strength of the MRI magnet is usually described in Tesla. The fields are typically between 0.2$T$ and 2.0$T$; the choice depends on the clinical application.

2.1.1 Permanent Magnet

With a field strength restricted to about 0.3$T$, the major advantage of a permanent magnet is running cost since no electrical generation is required. Its main disadvantages lie in its weight, which is normally huge so that the magnet must be mounted on a hardened floor. There is also the problem of the continuous loss of the magnetic field with time, and the homogeneity of the fields is relatively poor. It is, however, a good choice for low field machines where the permanent magnet MRI machine can be perpendicular to (or along )the axis of the body [3].

2.1.2 Resistive magnet

The magnetic field strength of the resistive magnet lies within the neighborhood of the permanent magnets (0.3$T$). Its major disadvantage is its running cost, since electricity is used to generate its magnetic field. Typically, its power consumption is between 100 – 120kw, and this restricts the size of the resistive magnet. Heat generated as a result of the resistance offered by the magnetic
windings can be removed by cooling with water. Its advantages over the permanent magnet are that it has a more homogeneous field, and can be switched off in case of an emergency.

2.1.3 Superconducting magnet

This is the preferred magnet type, since it does not suffer from any restrictions in size or field strength. Practically, there is no resistance to current flow, and hence no generation of heat from the electricity supply. This is a major advantage. The conducting magnets are constructed with alloys like niobium-tantalum and are divided into several sets to ensure magnetic homogeneity. The superconducting properties are obtained by immersing the alloy in liquid helium. The field current, usually up to 500A, continues when the supply is removed, thus minimising the cost of electricity. If the magnetic temperature rises just above the liquid helium temperature (4.2K), heat is transferred, and some helium boils off. The liquid helium is normally topped up after a specified period. If there is too little liquid helium left, the magnet will no longer be superconducting, but resistive. Under such circumstances, its temperature rises extremely rapidly, leading to a loss of the magnetic field. Any helium which is left, boils off and the helium vapor expands within moments, filling the MRI scanning room. Serious injury or death can result if everyone does not leave the room immediately. This is known as magnetic quench.

In order to achieve good image quality, it is essential that there is homogeneity in the magnetic field throughout the image volume. The preservation of magnetic field uniformity becomes a problem for increasing field strength. This primarily originates from external influences such as strong fields around power lines and electrical equipment, and/or ferromagnetic materials near the machine. It can also be a result of imperfections in the magnet’s construction [3]. These inhomogeneities can be corrected by shimming, a technique which helps to achieve a uniform magnetic field. Magnetic
field inhomogeneities are measured in parts per million (ppm), and can approach 100ppm even without external influence. Measurements must be made carefully since small imperfections can change the $T_2^*$ relaxation time. The effect of small non-uniformity, after correction, can be reduced by increasing the gradient fields.

### 2.2 Angular momentum of the proton

We will be looking at the relation between the proton intrinsic angular momentum (also referred to as spin) and its moment. The protons and neutrons that make up the constituents of the nuclei, each have a spin-$\frac{1}{2}$ in units of $\hbar(\frac{1}{2\pi})$, and as such are examples of a class of particles known as fermions. A nucleon possesses an orbital angular momentum, $L$, and a spin angular momentum, $S$. These two angular momenta sum up vectorially to give the total angular momentum, $J$ [6].

\[
J = L + S
\]

The vectorial sum of the angular momenta of the constituent nucleons in a given nucleus therefore defines the nuclear spin. The spin quantum number varies with the number of protons, $Z$, and neutrons, $N$. This vector has an absolute magnitude given by:

\[
|J| = \sqrt{J(J + 1)\hbar}
\]

It is integral for nuclei of even mass number and half integral otherwise.

For a nucleus with an even number of both neutrons and protons, there is no net spin. This is due to Pauli’s exclusion principle, which dictates that pairing allows for a net cancellation of spin. There are 165 isotopes in this class, but these are irrelevant to MRI since it deals with the magnetic properties of nuclei [7]. If otherwise, there will be a spin associated with the proton. As a result of this spin, there is an associated magnetic dipole moment. Even though this is very small relative to that of an electron, it still plays a vital role in the theory of nuclear structure. Surprisingly, the uncharged neutron has a magnetic moment and this reflects the presence of an underlying quark-substructure: the charged components responsible for this phenomenon [6]. The key to MRI is based upon the behaviour of these particles in electromagnetic fields.

The nuclear magnetic moment, unlike angular momentum, is not quantised and is not governed by any conservation laws. It is described by a vector $\mu_j$ and can be expressed in nuclear magnetrons, defined:

\[
\mu_n = \frac{e\hbar}{2m_p}
\]


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The nuclear gyromagnetic ratio is defined as

$$\gamma = \frac{\mu}{Jh}$$  \hspace{1cm} (2.4)

$$\Rightarrow \mu = \gamma Jh$$  \hspace{1cm} (2.5)

Thus, a nucleus having net spin, $J$, produces a magnetic moment $\mu$.

Classically, this is not perfectly true since spin is a quantum property. This description only helps us to have an insight into what spin is; there is no proper quantum description of it. The spinning proton can be thought of as a tiny bar magnet with north and south poles, i.e. a magnetic dipole. The magnetic moment of a proton is very small and indeed undetectable, we can represent the cumulative magnetic field of many protons as a single vector. There are about $10^{21}$ protons per cubic centimeter of tissue [8]. In the absence of a magnetic field, the protons are randomly distributed within the sample as a result of thermal agitation, see Fig (2.3). Hence, the magnetic fields of the nuclei cancel out, and no net nuclear magnetic properties are exhibited by the tissue. However, we have seen from the first chapter that when tissue is subjected to a strong magnetic field with field strength, $B_0$, in the Z-direction, the protons magnetic dipoles tend to align with the magnetic field, adopting one of the ranges of possible values of $J_z = J, J - 1, \ldots, -(J - 1), -J$ [7].

The relationship between the component of the spin of the nucleus along that axis, and its magnetic moment in that direction is given by equation (2.5): for the Z-direction it will be more convenient to write

$$\mu_z = \gamma h J_z$$  \hspace{1cm} (2.6)

The individual protons align such that they produce a net magnetisation vector in the direction along which the field lies, the Z-direction, see Fig. (2.4).

The net spin, $J$, produces a magnetic moment $\mu$, which precesses around the axis of the external

Figure 2.2: A single proton with spin, produces a magnetic moment, $\mu$
Figure 2.3: Protons with dipole magnetic fields, randomly spread out in the absence of an external magnetic field.

Figure 2.4: The individual protons are aligned, such that they produce a net magnetisation in the direction of the applied magnetic field (z-direction).
magnetic field $B_0$. Its precessional frequency is given by $\omega_0$. The vector sum of all the protons precessing about the axes across the allowed range of possible values is a vector aligned with $B_0$, see Figure (2.5). There is no magnetisation field perpendicular to the applied magnetic field axes at equilibrium because of the random precessional phases of the proton magnetic dipoles.

Each spin quantum state has a nuclear energy, and their collective energy levels define the Zeeman energy levels [7]. The spin quantum states are therefore indistinguishable in the absence of an external magnetic field; all spin states have the same Zeeman energy level.

In an external magnetic field of strength $B_0$, the energy corresponding with a certain Zeeman level is given by:

$$E = -\mu B_0 = -\gamma \hbar J B_0$$  \hspace{1cm} (2.7)

For a particular spin quantum level along the $Z$-direction, the energy is

$$E_z = -\gamma \hbar B_0 J_z$$  \hspace{1cm} (2.8)

where $J_z$ is one of the allowed spin states. This implies that an alteration of $B_0$ can thus change the spin energy but the distribution of the $J_z$ spin levels is unaltered. The nuclear magnetic dipoles align themselves at larger angles from the external magnetic field at increasing Zeeman energy levels. Thus, a stronger magnetic field only causes better alignment of the nuclear magnetic dipole along its magnetic field direction, leaving the entire distribution of the Zeeman energy level of the sample unchanged. To change this distribution will require the spin energies of the nuclei to be
increased by an amount defined by $J$. Thus, the absorption of photons with specific energies is required. The least such energy is defined by the angular frequency $w_0$ of the photon and is given by

$$E_p = \hbar w_0$$

(2.9)

where $\hbar = \frac{\hbar}{2\pi}$

If the energy required for a spin state to be excited to a higher state is $E_p$, this will imply that $E_p$ is the photon energy that leads to its absorption. Thus,

$$\hbar w_0 = \gamma \hbar B_0 (J_f - J_i)$$

(2.10)

Here, $J_f$ and $J_i$ signify the final and initial spin states respectively, and $\gamma$ the gyromagnetic ratio unique to each element.

For our assumption of a unit difference between the two spin states, we will get

$$\hbar w_0 = \gamma \hbar B_0 \Rightarrow w_0 = \gamma B_0$$

(2.11)

Hence, a photon with frequency, $w_0$, can excite a nucleus from a spin state to the next adjacent spin state. This frequency $w_0$, is known as the Larmor frequency. This equation can also be written in terms of linear frequency as

$$f = \frac{\gamma}{2\pi} B_0$$

(2.12)

From the classical mechanical standpoint, an external field $B_0$, will tend to rotate the nuclear dipole of a nucleus of non-zero spin and magnetisation $\mu_0$, with a torque, $\tau$, to an alignment with itself. Thus:

$$\tau = \mu \times B_0$$

(2.13)

since

$$\mu = \hbar J \gamma$$

(2.14)

and

$$B_0 = \frac{\omega_0}{\gamma}$$

(2.15)

Then,

$$\mu \times B_0 = \hbar J \gamma \times \frac{\omega_0}{\gamma}$$

(2.16)

and since $\gamma$ is constant,

$$\mu \times B_0 = \hbar J \times \omega_0$$

(2.17)

But $\omega_0$, which is the angular rate of precession aligned with the external magnetic field, can be
written as:
\[ \omega_0 = \frac{d\theta}{dt} \]  

(2.18)

\( \theta \), is the precession angle between the nuclear magnetic dipole and the external magnetic field. Now consider two states of the precessing nuclear magnetic dipole, \( \mu_1 \) and \( \mu_2 \), as in the Figure (2.6), and representing instants separated by a time \( \delta t \). Now by equation (2.14), we have \( \mu = h\gamma J \), so:

\[ \delta \mu = \mu_2 - \mu_1 = h\gamma (J_2 - J_1) = h\gamma (\delta J) \]

(2.19)

Dividing both sides by \( \delta t \):

\[ \frac{\delta \mu}{\delta t} = \frac{h\gamma}{\delta t} \frac{\delta J}{\delta t} = \frac{h\gamma (J \sin \phi) \delta \theta}{\delta t} \]

from the tip of \( \mu_1 \), to the tip of \( \mu_2 \)

(2.20)

but this is simply \( \gamma hJ \times \omega_0 \) in the limit \( \delta t \to 0 \), hence:

\[ \frac{d\mu}{dt} = \gamma hJ \times \omega_0 \]

(2.21)

and by comparison with equation (2.17):

\[ \frac{d\mu}{dt} = \gamma (\mu \times B_0) \]

(2.22)

Equation (2.21) shows the change in the magnetic moment with respect to time \( t \). The change is orthogonal to the plane containing both \( \mu \) and \( B_0 \). This fundamental equation of motion is at the
heart of the rotations and precessions that we shall frequently discuss.

The total distance, $D$, covered in a complete rotation is given by the circumference of the circular path. Therefore,

$$D = 2\pi r = 2\pi |\mu| \sin\phi$$  \hspace{1cm} (2.23)

From equation (2.21), we already have our speed. We use this to compute the time, and get

$$T = \frac{2\pi |\mu| \sin\phi}{\gamma(\mu \times B_0)} = \frac{2\pi |\mu| \sin\phi}{\gamma |\mu||B_0| \sin\phi}$$  \hspace{1cm} (2.24)

$$T = \frac{2\pi}{\gamma B_0} \Rightarrow \frac{1}{T} = \frac{\gamma B_0}{2\pi}$$  \hspace{1cm} (2.25)

$$\omega_0 = \gamma B_0$$  \hspace{1cm} (2.26)

This reproduces equation (2.11) for a photon of frequency, $\omega_0$, responsible for nucleic excitation as discussed earlier.

To generate photons of the correct frequency, an alternating current is passed through a simple conducting wire loop in the direction perpendicular to the axial magnetic field. The time for which the nuclei are exposed to this perpendicular magnetic field determines the number of spin states that will adopt more energetic spin levels. We can thus set the angle of the total magnetisation vector with respect to the external magnetic field by our choice of exposure time [7].

### 2.3 Generation of Nuclear magnetic resonance signal

The behaviour of the net nuclear magnetic moment of the materials can usually be sufficiently described using classical theory of magnetism. We shall be considering a proton for our discussion. It possesses a charge $+e$ and has a spin $J$. When an electric charge circulates through a conducting loop, it produces a magnetic field. The charge on the proton can be considered as being distributed and rotating about a central axis as a result of its spin. This gives rise to a field and a magnetic moment. In a sample comprising many nuclei, the net magnetic moment $\vec{M}$, can be derived from the vector sum of all nuclear magnetic moments. In a static magnetic field $B_0$, the net magnetic moment at equilibrium will be aligned to $B_0$. In order that $\vec{M}$ can be measured, it must be tipped away from the $z$-axis where $B_0$ lies. This is the basis of NMR measurements. If another magnetic field of a different strength, rotating at the same frequency is applied in the $xy$ plane, $\vec{M}$ will experience a second rotational force, its angle depending on the strength of this new applied magnetic field. This magnetic field has frequency in the radio frequency (RF) range for NMR measurements and a 90 degree RF pulse converts $\vec{M}$ into the $xy$ plane, such that the longitudinal magnetisation $M_L = 0$ When this RF pulse is removed, a typical signal known as the free induction decay (FID) is obtained [9]. In the Imaging of the human system, this technique is used to obtain high quality images and is referred to as Magnetic resonance imaging.
2.4 MRI concepts

Here, we introduce the basic elements that comprise MRI. It will exclude derivations and details of results as they will be discussed in the later chapters.

We already know that NMR relies upon the interaction between a nuclear spin with an external magnetic field, $B_0$. The spin axes of nuclei having net spins, precess about the north-south axis of the external magnetic field.

The precession of the proton is the circular motion of its axis of rotation about another fixed axis, caused by the application of a torque in the direction of the precession [10]. We notice that the precessional angular frequency for the magnetic moment of the proton is:

$$\omega_0 = \gamma B_0$$  \hspace{1cm} (2.27)

$\gamma$ is a constant called the gyromagnetic ratio of the hydrogen proton, and has a value of roughly $2.68 \times 10^8$ rads/s/Tesla in water. This frequency is also referred to as the Larmor frequency.

2.4.1 Magnetic resonance imaging

We aim to correlate acquired signals with the spatial location of their various sources. If a magnetic field which has a gradient along the x-axis is applied to the sample, the frequency of the NMR signal from the sample is given by:

$$\omega(x) = \gamma B(x)$$  \hspace{1cm} (2.28)

where $x$ denotes the positions in the direction of the gradient of the field. The signal frequency thus depends on the x position, hence, it is possible that the signal strength can be mapped spatially to produce an image. A Fourier transform is used to invert the signal.

2.4.2 Relaxation times

We have so far, talked about how the signals are generated, but have left an important point up to this stage. The spin-lattice decay of the signal as a result of the interaction between the spin and its surrounding, is an important consideration in the signal acquisition process. This is characterised by a relaxation time known as $T_1$. After the magnetisation vector has been rotated to the transverse plane, and the tipping field has been switched off, it begins to realign itself with the direction of the static magnetic field. The time of regrowth is characterised by the constant $T_1$, and is called the spin-lattice or longitudinal relaxation time. It is influenced by the interaction of each nuclear spin with its atomic neighborhood. The magnetisation time evolution is described by the Bloch equation, which takes into account the relaxation and precession effect [10]. For $M_T = 0$, 

the subsequent regrowth of $M_L$ is:

$$M_L(t) = M_0(1 - \exp\left(-\frac{t}{T_1}\right)) \quad (2.29)$$

Another relaxation effect, is the dephasing of the precession of the nuclear magnetic dipole representing “spin-spin” decay of the transverse magnetisation. We consider an experiment where the 90 degree RF pulse is applied at an interval of time $T_R$, the previous transverse magnetisation has decayed due to the spin-spin effect, and the only magnetisation present is the longitudinal, which is rotated into the transverse plane. If we sample the signal information at time $T_E$ (echo time) following the pulse, the signal magnitude is proportional to the transverse magnetisation and is given by

$$M_L(T_E) = M_0(1 - \exp\left(-\frac{T_R}{T_1}\right)\exp\left(-\frac{T_E}{T_2}\right)) \quad (2.30)$$

The term $\exp\left(-\frac{T_E}{T_2}\right)$ represents the spin-spin decay factor characterised by $T_2$, which is caused by the dephasing of the nuclear precession as a result of variations in local precessional frequencies [11].

Generally, there is extra suppression from external magnetic field inhomogeneities, which leads to a relaxation time $T_2^*$ that is shorter than $T_2$ i.e $T_2^* < T_2$. This effect can be corrected for by rephasing or echoing of the source of dispersion and has been assumed in the above equation. The idea of this correction is to flip all the nuclear magnetic dipoles 180 degrees in the transverse plane. The dephasing is reversed, refocusing the dispersion of any external field at the time of echo, $T_E$.

$T_1$, $T_2$ and spin density vary in tissues and constitute the essential aspect of MRI.

### 2.4.3 Contrast

Another aspect of MRI is the image resolution, which is a measure of the size of spatial features which can be distinguished. Image resolution is independent of the wavelength of the RF field, but depends on the way the signal and noise are sampled and filtered. The high sensitivity of MRI to proton density, relaxation times, proton motion, and temperature, in tissues has made it a useful tool in differentiating between materials. This set of variables allow images to be reconstructed with different contrast levels depending on the choice of image, making MRI more vast than many other techniques, which mainly have one type of contrast.

### 2.5 Resonance

When an RF pulse of strength $B_1$ is used to tip the net sample magnetisation, the sample is exposed to photon from an RF transmitter at a frequency which is equal to the precessional frequency of the protons. Resonance occurs when the energy of the RF photons equals the difference in energy
Figure 2.7: The longitudinal magnetisation, $M_z$, shown in the z-direction, and equilibrium magnetisation maximal in the longitudinal direction of the sample, but shown displaced from the z-axis as a result of transverse magnetisation.

The photon energy is given by:

$$E = hf$$  \hspace{1cm} h \text{ is the planck’s constant, and } f \text{ the frequency of the photon} \tag{2.31}$$

The RF amplitude and duration determines the number of nuclei that will transit to a higher energy level; a greater number of transits when the amplitude or duration is increased. If this period is long enough, the longitudinal magnetisation experiences a change in magnitude from a maximum equilibrium value, through zero, to a maximum negative value [8]. Quantum mechanically, the system is viewed from a discrete energy standpoint, and rather than consider RF pulses as waves, they are seen as discrete photons. Here, the discrete energy gap between two spin states is dependent on the magnetic field strength [8], so that when $B_0$ is increased, the energy gap increases in equal proportion. When spins from lower energy states move to higher energy states, the system absorbs energy. This is only true when an exact energy is applied, otherwise, there will be no excitation.
It corresponds to a specific frequency which is the precessional frequency of the protons. The RF pulse amplitude determines the tipping angle of the system [10].

The tipping angles show the rotational characteristics of the longitudinal magnetisation vector [3]. They are usually either 90 or 180 degrees, but can sometimes be chosen to be smaller or larger to display different tissue contrasts suitably. The duration for a particular tip is linearly related to the displacement angle: $\theta = \omega t$. This implies that a smaller angle often yields faster MR imaging.

The MR signal, as will be discussed in the next chapter, originates from a magnetisation in the transverse plane, $M_T$. We shall see that a tip of 90 degrees yields the greatest number of signals relative to other tipping angles.
Chapter 3

MAGNETISATION VECTOR, BLOCH EQUATION, PULSE SEQUENCES AND MANIFESTATION OF CONTRASTS

3.1 Magnetisation vector

Magnetisation, $\vec{M}$, is the local magnetic moment per unit volume, of the protons in a sample (tissue). We consider a volume element, $V$, small enough for there to be a constant external magnetic field over it, and which contains a large number of protons. The magnetisation is defined as

$$\vec{M} = \frac{1}{V} \sum_{\mu} \vec{\mu}_i$$  \hspace{1cm} (3.1)

The set of spins in $V$ can be seen as an ensemble of spin with phase coherence. We wish to sum over the equation of motion derived in equation (2.22). Neglecting the interaction of each proton with its environment, we get

$$\frac{1}{V} \sum_i \frac{d\mu_i}{dt} = \gamma \sum_i \mu_i \times B_0$$  \hspace{1cm} (3.2)

$$\Rightarrow \frac{d\vec{M}}{dt} = \gamma \vec{M} \times B_0$$  \hspace{1cm} (3.3)

Note that equation (3.3) is only valid for non-interacting protons. We shall attempt to under-
stand the magnetisation and their time differentials in terms of the longitudinal and transverse components

\[ M_L = M_z \]  \hspace{1cm} (3.4)

and

\[ M_T = M_x i + M_y j \]  \hspace{1cm} (3.5)

respectively.

From equation (3.3), when we evaluate the corresponding components of \( M_L \) and \( M_T \), we get

\[ \frac{dM_L}{dt} = 0 \quad \text{because} \ M_L \text{ is in the same direction as } B_0 \]  \hspace{1cm} (3.6)

and

\[ \frac{dM_T}{dt} = \gamma M_T \times B_0 \]  \hspace{1cm} (3.7)

These two equations are true only for non-interacting protons. However, since of interest to us is the interaction of protons with other protons and also with their surroundings, an additional term which depends on delay parameters will be introduced to account for the interactions already mentioned. These terms are different for the two equations. This difference is due to the fact that contrary to a given magnetic moment, the magnitude of the magnetisation is not constant (it is a vector sum of proton spins). The \( M_L \) and \( M_T \) components relax differently as they approach equilibrium value.

### 3.1.1 Spin-lattice relaxation time, \( T_1 \)

After the introduction of a static magnetic field, \( B_0 \), the protons are aligned as much as possible in its direction. An alternating magnetic field with a much lower strength is then introduced in the transverse plane and causes an excitation of the spin states.

When the transverse magnetic field is removed, there seems to be a regrowth of the longitudinal magnetisation. A constant interaction growth rate from the proton interaction with the lattice suggests that the rate of change of longitudinal magnetisation, \( \frac{dM_L}{dt} \), at any point in time, is proportional to the difference, \( (M_0 - M_L) \) measured at that same instant [10]. This constant of proportionality is determined empirically and is found to be the inverse of the growth rate. That is

\[ \frac{dM_L}{dt} = \frac{1}{T_1}(M_0 - M_L) \]  \hspace{1cm} (3.8)
where $T_1$ is the spin-lattice relaxation time.

This relation encompasses the general behavior of the interacting protons and its surrounding. It is a more realistic model for our description. Typical values for various tissues are shown in the table below:

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$T_1$ (ms)</th>
<th>$T_2$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray matter (GM)</td>
<td>950</td>
<td>100</td>
</tr>
<tr>
<td>White matter (WM)</td>
<td>600</td>
<td>80</td>
</tr>
<tr>
<td>Muscle</td>
<td>900</td>
<td>50</td>
</tr>
<tr>
<td>Cerebrospinal fluid (CSF)</td>
<td>4500</td>
<td>2200</td>
</tr>
<tr>
<td>Fat</td>
<td>250</td>
<td>60</td>
</tr>
<tr>
<td>Blood</td>
<td>1200</td>
<td>100-200</td>
</tr>
</tbody>
</table>

Table 3.1: Representative values of $T_1$ and $T_2$, in milliseconds, for hydrogen components of different human body tissues, at $B_0 = 1.5$ Tesla, and 37 degrees Celsius.

From equation (3.8),

$$\frac{dM_L}{M_0 - M_L} = \frac{dt}{T_1}$$  \hspace{1cm} (3.9)

$$- \ln(M_0 - M_L) = \frac{t}{T_1} + C$$  \hspace{1cm} (3.10)

$$M_0 - M_L = Ce^{\frac{t}{T_1}}$$  \hspace{1cm} (3.11)

At $t = 0$,

$$C = M_0 - M_L$$  \hspace{1cm} (3.12)

Thus,

$$M_L(t) = M_0 - (M_0 - M_z(0))e^{\frac{t}{T_1}}$$  \hspace{1cm} (3.13)

$$M_L(t) = M_L(0)e^{\frac{t}{T_1}} + M_0(1 - e^{\frac{t}{T_1}})$$ \hspace{1cm} where $M_L = M_z$  \hspace{1cm} (3.14)

This implies that after the application of the RF pulse, the longitudinal magnetisation displays an exponential form, showing an evolution from $M_L(0)$ to the equilibrium value $M_0$. This is the key to understanding the regrowth after initial displacement of $M_L$. See Fig (3.1) for the regrowth diagram.

For an initial situation where $M_L = 0$ (i.e for the condition achieved when an RF pulse of 90 degrees
MAGNETISATION VECTOR, BLOCH EQUATION, PULSE SEQUENCES AND MANIFESTATION OF CONTRASTS

Figure 3.1: The net sample magnetisation along the axis of the static magnetic field increases as nuclei return to lower spin level

is applied), the subsequent regrowth of $M_L$ becomes,

$$M_L(t) = M_0(1 - e^{-\frac{t}{T_1}})$$

(3.15)

When $t = T_1$, then $(1 - e^{-1}) = 0.63$ and $M_L = 0.63M_0$

The time for a complete recovery is directly dependent on the $T_1$ time constant and exponential regrowth. For example, for time equals $3 \times T_1$ after the RF pulse, 95 percent of the equilibrium magnetisation is established. After about $5 \times T_1$, it is seen that a full longitudinal magnetisation is achieved [8].

To determine the $T_1$ time of a specific tissue or material, an initial 90 degrees pulse, which reduces the longitudinal magnetisation to zero, is followed by a delay time, $\tau$, and then another 90 degree pulse is applied to determine the regrowth by displacing it out of the transverse plane. If this is done with several delay times, and a plot of signal versus $\tau$ is got, $T_1$ can be calculated from a curve (since it is the time taken to regain 63 percent of the signal) fitted with a suitable function.

$T_1$ assumes different values for different tissues and pathologies of tissues. It depends on the interaction of excited spins with the surrounding molecular lattices.
3.1.2 Spin-Spin relaxation time, $T_2$

The spins experience local magnetic fields, which are combinations of the applied fields and fields of their neighboring nuclear magnetic dipoles, and these constitute an important mechanism for the transverse decay. This variation in local fields leads to differences in precessional frequencies of the protons, thus making them dephase with time.

Since $\gamma M \times B_0$ is always perpendicular to $M_T$, it follows that $\gamma M \times B_0$ describes a continuous change in the direction of $M_T$, but not its magnitude, i.e $\gamma M \times B_0$ describes the rotation of $M_T$. If one now looks only at the magnitude of $M_T$, and plots it against time, then it turns out that it is an exponential decay curve such that:

$$\frac{dM_T}{dt} = -\frac{M_T}{T_2} \quad \text{magnitude only} \tag{3.16}$$

where $T_2$ is chosen to give the right curve. Combining the changes in direction and magnitude, we have:

$$\frac{dM_T}{dt} = \gamma M_T \times B_0 - \frac{1}{T_2} M_T \tag{3.17}$$

This leads to an exponential decay of any initial value for $M_T$. The differential equation has a standard decay rate of the form

$$\frac{dM_T}{dt} = -\frac{1}{T_2} M_T \tag{3.18}$$

and the solution to this is

$$M_T(t) = M_T(0)e^{-\frac{t}{T_2}} \tag{3.19}$$

In the presence of inhomogeneities due to external magnetic fields, $M_T$ decays faster than is a perfectly homogeneous field. The resulting time constant is called $T^*_2$. The signal falls exponentially with the $T^*_2$ relaxation time constant. $T^*_2$ is dependent on the homogeneity of the main magnetic field. The effects of inhomogeneities can be minimised using the spin-echo techniques discussed in section (3.31).

3.2 Bloch equations

The differential equations for the transverse and longitudinal magnetisation, and the relaxation terms in the presence of a magnetic field, can be coupled to get an empirical vector equation referred to as the Bloch equation. This is shown below:
Figure 3.2: The decay of magnitude of the transverse magnetisation from an initial value

\[
\frac{dM}{dt} = \gamma M_T \times B_0 + \frac{1}{T_1}(M_0 - M_L) - \frac{1}{T_2}M_T
\]  

(3.20)

Let us calculate the components of the cross product, assuming \( B_0 \) is the Z-direction. This produces the three components of the equation as:

\[
\frac{dM_z}{dt} = \frac{M_0 - M_L}{T_1}
\]  

(3.21)

\[
\frac{dM_x}{dt} = \omega_0 M_y - \frac{M_x}{T_2}
\]  

(3.22)

\[
\frac{dM_y}{dt} = -\omega_0 M_x - \frac{M_y}{T_2}
\]  

(3.23)

Equation (3.21) is the same as equation (3.8). For the differentials of the x and y components shown in equations (3.22) and (3.23), the relaxation terms can be easily eliminated by a change of variable: \( M_x = m_x e^{-\frac{t}{T_2}} \) and \( M_y = m_y e^{-\frac{t}{T_2}} \). Equation (3.21) becomes:

\[
\frac{dm_x}{dt} = m_y \omega_0
\]  

(3.24)

and equation (3.22),

\[
\frac{dm_y}{dt} = -m_x \omega_0
\]  

(3.25)
These can be written as a system of equations in matrix form as:

\[
\begin{pmatrix}
\dot{m}_x \\
\dot{m}_y
\end{pmatrix} =
\begin{pmatrix}
0 & \omega_0 \\
-\omega_0 & 0
\end{pmatrix}
\begin{pmatrix}
m_x \\
m_y
\end{pmatrix}
\]

The the solution to the system is:

\[
m_x = m_x(0) \cos \omega_0 t + m_y(0) \sin \omega_0 t
\]

\[
m_y = m_y(0) \cos \omega_0 t + m_x(0) \sin \omega_0 t
\]

This implies,

\[
M_x(t) = e^{\frac{-t}{T_2}} (M_x(0) \cos \omega_0 t + M_y(0) \sin \omega_0 t)
\]

\[
M_y(t) = e^{\frac{-t}{T_2}} (M_y(0) \cos \omega_0 t + M_x(0) \sin \omega_0 t)
\]

\[
M_z(t) = M_z(0)e^{\frac{-t}{T_1}} + M_0(1 - e^{\frac{-t}{T_1}})
\]

The asymptotic solution to this equation defines the equilibrium or steady state of the equation, and is \(M_x(\infty) = M_y(\infty) = 0\), and \(M_z(\infty) = M_0\).
Comparison between $T_1$, $T_2$ and $T_2^*$

The characteristic mechanisms relevant to the relaxation processes, $T_1$ and $T_2$, occur differently, depending on their various environments, and also the tissue. In MRI, high contrast is achieved because of the differences between $T_1$, $T_2$ and $T_2^*$, and also proton density differences. $T_1$ is always greater than $T_2$. In cases where the local magnetic field is disrupted by some agents such as paramagnetic blood degradation products, elements with unpaired electron spins, or ferromagnetic materials, there is a decrease in $T_2^*$ decay time. The order in magnitude of the different relaxation times can be written thus: $T_1 > T_2 > T_2^*$, and does not depend on the tissue type in which the spins are found.

3.3 Spin echo, inversion recovery, and magnetic gradients

To obtain excellent contrast sensitivity, we must emphasize the differences in spin density, as well as the $T_1$ and $T_2$ relaxation time constants of different tissues. The timing, order, polarity and number of repetitions of the RF pulses, and the applied gradient, make the signal dependent on these relaxation characteristics [12].

Spin echo and inversion recovery sequences are techniques used in MRI to achieve suitable contrast of tissues. We can also obtain desired images with characteristic “contrast weighting” using these in conjunction with other localisation methods (an example is to influence $T_1$ dependencies with respect to $T_2$).

3.3.1 Spin echo

For spin echo contrast mechanism, a steady state situation is required. This is reached by applying a series of 90 degree pulses separated by a period known as repetition time, $T_R$. These pulses convert the longitudinal magnetisation into the transverse plane. The delay time (repetition time) is always insufficient to allow for complete relaxation onto the longitudinal plane, therefore, lesser signals are observed for subsequent pulses. Mostly, the system is found to saturate after the application of the third pulse and is, at this point, said to be in a steady state. At the steady state, the longitudinal magnetisation is at equilibrium and does not change from pulse to pulse.

Between the 90 degree pulses, spin-spin interaction causes decay of transverse magnetisation signal, according to the $T_2$ time of each tissue. After time, $\frac{T_E}{2}$, a 180 degree pulse causes an inversion of the spin system, and refocuses the transverse magnetisation (this is the formation of an echo at a time of echo). The inversion of the spin system cancels the external magnetic field inhomogeneities, since precession is in the opposite direction. Therefore, the maximal echo amplitude depends solely on $T_2$ ($T_2^*$ effect vanishes) and the proton density of the tissue.

A mathematical explanation for the process is given below. We shall assume that the magnetisation
of the tissue is tipped by the first pulse. Suppose this happens immediately at \( t = 0 \), such that they point along the \( y \)-axis. The spins at various positions, \( \mathbf{r} \), begin to dephase relatively, as they experience different field strengths not exactly equal to \( B_0 \). The accumulated phase of the spin at \( \mathbf{r} \) in the rotating frame is:

\[
\theta(r, t) = -\gamma \Delta B(r)t
\]

where \( \theta = \frac{\omega}{\gamma} \). When the 180 degree RF pulse is applied along the \( y \)-axis at the time \( t_1 \), the spins which had previously accumulated extra phase now have the negative of that phase, which is written as:

\[
\theta(r, t_1^+) = -\theta(r, t_1^-) = \gamma \Delta B(r)t_1
\]

The spins continue, after time \( t_1 \), to accumulate phase according to equation (3.31):

\[
\theta(r, t) = \theta(r, t_1^+) - \gamma \Delta B(r)(t - t_1) = -\gamma \Delta B(r)(t - 2t_1)) = -\gamma \Delta B(r)(t - T_E) \quad t > t_1
\]

with the echo time given by \( T_E = 2t_1 \). The rate at which phase is accumulated by each spin is unchanged, therefore all the spin will return to \( \theta = 0 \) at the same time. Equation (3.33), shows that the accumulated phase of all the spins experience a time-independent field variation, and will get back to equilibrium at \( t = T_E \). This suggests that all spin will realign at the same time. This is called spin-echo [10].

**Spin-echo pulse sequence**

The spin-echo sequence is described by the two timing parameters \( T_R \) and \( T_E \) produced by the machine. It is this selection that allows for the contrast weighting of the sample. Fig (3.4), shows a spin-echo diagram depicting the sequence of the RF pulsing and magnetic gradient application used to localise the returning echo from a given slice of tissue (Details of this localisation process will be discussed shortly.)
Figure 3.3: A spin echo pulse sequence, starting with the 90 deg. pulse which causes a transverse magnetisation, followed by a loss of phase coherence according to $T_2^*$. After a time, $T_E$, a 180 deg. pulse is applied, which reverses the direction of the dephased spin, canceling the effect of $T_2^*$ and also rephasing the spins.
Figure 3.4: A generic spin echo pulse sequence, showing RF pulsing and magnetic gradient application.

**$T_1$ weighting**

This emphasizes the $T_1$ characteristics of tissues, while de-emphasizing the influences caused by the $T_2$ relaxation time or spin density. A relatively short $T_R$, which maximizes the differences between longitudinal magnetisation in target tissues during return to equilibrium is used. A short $T_E$, which minimizes $T_2$ effect during signal acquisition is used.

**Spin density weighting**

This depends on the differences in the number of magnetisable protons in a volume of tissue. Tissues with greater proton density achieve larger longitudinal magnetisation at equilibrium. Thus, tissues with much hydrogen content like lipids, have a greater spin density than their proteinaceous counterparts. For a minimal $T_1$ difference between tissues, a long $T_R$ is used. This permits the reformation of a large amount of longitudinal magnetisation, such that differences in the transverse magnetisation observed at the time of the 90 degree pulse are mainly due to spin density. A very short $T_E$ will help to preserve the signal amplitude differences during acquisition of echo, so that just a little $T_2$ decay is made manifest. This results in the strongest signal, with a high signal-to-noise ratio, producing a slight tissue contrast with a relatively poor image contrast.

**$T_2$ weighting**

The emphasis is a reduction of the $T_1$ effect by using long $T_R$, and the manifestation of $T_2$ effects by using long $T_E$ (transverse decay with longer $T_E$). This is accomplished as the acquisition of a second echo from the spin echo pulse sequence emphasizing spin density weighting. However,
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3.3.2 Inversion recovery

The inversion recovery pulse sequence is more sensitive to $T_1$ relaxation times of tissues. It is similar to spin-echo, but with an initial 180 degree pulse applied. This pulse is defined to occur at a time interval $T_I$ and inverts the longitudinal magnetisation. The decay of the magnetisation between the pulses yields a signal strongly dependent on $T_1$. The initial 180 degree pulse can be thought of as exciting the sample’s magnetic moment into the antiparallel direction, with the same magnitude but negative polarity [10]. The time for recovery due to this pulse is longer than the 90 degree tip seen in the spin-echo.

After a decay time $T_I$, a 90 degree pulse is applied to rotate the fraction of spin already longitudinally realigned, into the transverse plane, so that signal can be produced. For signal acquisition, a second 180 degree pulse at a time, $\frac{T_E}{2}$, creates an echo signal that appears at time, $T_E$. The amplitude of the echo is found to be solely dependent on $T_1$, $T_E$, $T_I$ and $T_R$. $T_R$ is the time between 180 degree pulses.

Measurement of $T_1$

For an accurate measurement of $T_1$ through inversion recovery, a track of the longitudinal compo-
MAGNETISATION VECTOR, BLOCH EQUATION, PULSE SEQUENCES AND
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ponent during the period of 180 and 90 degree pulses is kept. The longitudinal magnetisation after
the initial 180 degree pulse is negative and is written as:

\[ M_L(0^+) = -M_0 \] (3.34)

We define \( t = T_I \) as the time for the application of the 180 degree pulse. Recall equation (3.14):

\[ M_L(t) = M_L(0)e^{\frac{t}{T_1}} + M_0(1 - e^{\frac{t}{T_1}}) \] (3.35)

From this, the magnetisation regrowth to its equilibrium value in the interval between pulses is:

\[ M_L(t) = -M_0e^{\frac{t}{T_1}} + M_0(1 - e^{\frac{t}{T_1}}) = M_0(1 - 2e^{\frac{t}{T_1}}) \quad 0 < t < T_1 \] (3.36)

After the application of the 90 degree pulse to tip the magnetisation into the transverse plane, so
as to provide the initial signal, the transverse magnetisation evolves as:

\[ M_T = |M_0(1 - 2e^{\frac{-T_I}{T_1}})| \quad e^{-(t-T_I)}, \quad t > T_I \] (3.37)

As already mentioned the magnitude of the signal is modulated by the \( T_1 \) dependent factor, \(|M_0(1 - 2e^{\frac{-T_I}{T_1}})|\). For a repeated spin-echo experiment, this can be compared with \(|(1 - e^{\frac{-T_E}{T_1}})|\). The factor 2 in the equation exists because the longitudinal magnetisation must recover from a maximum negative \(-M_z\) to a maximum positive \(M_z\), instead of from 0 to \(M_z\) as in the spin-echo with a 90
degree pulse. The inversion time, \(T_I\) controls the difference in contrast possible between tissues,
and \(T_E\) is kept as short as possible to maximise the signal amplitude and minimise \(T_2\) effects [8].

Generally, the inversion recovery experiment consists of a combination of two pulses. The first,
inverts the longitudinal magnetisation. The second, tips the longitudinal magnetisation into the
transverse plane, so that the signal can be measured. On the other hand, the spin echo inversion
recovery experiment consists of three RF pulses, which are the two as stated above and a third,
which inverts both the longitudinal and transverse magnetisation leading to an echo.

3.3.3 Magnetic gradient

MRI signal frequency is directly related to the magnetic field strength as described by the Larmor
equation. If the strength of a magnetic field varies with location in a single direction, i.e there is
a linear magnetic gradient along the axis, then the Larmor frequency will also vary linearly with
position in this direction. \(G_x, G_y\) and \(G_z\) are used to denote these three gradients, which enable
the signal from the patient to be spatially encoded. The gradient field strengths are changed in a
particular sequence in order to produce image data signals. Gradient coils are specially constructed
and placed within the main magnet. They serve the three axes covering the whole of the patient.

-A frequency encoded z-axis (slice selection)
-A phase encoded y-axis horizontally opposed to the z-axis.

-A frequency encoded x-axis vertically opposed to the z-axis.

**Slice selection gradient** $G_z$

If a slice select gradient, $G_z$, is superimposed on a main field, $B_0$, there is a small linear increase of the field strength over the length of the gradient and the Larmor frequency for the proton will vary along the gradient direction $z$, as:

$$\omega = \gamma (B_0 + Gz)$$

(3.38)

In the slice selection direction $z$, the proton nuclear magnetic dipole will precess with a frequency, $\omega$ such as:

$$z = \frac{\omega - \gamma B_0}{\gamma G}$$

(3.39)

The protons found outside the slice portion, $z$, will be unaffected, therefore there will be no emission of signal. This explains that we can localise a region of desired interest and get spatial signal information [3]. The main field must be perfectly uniform, otherwise variations from gradient fields can be lost due to inhomogeneity in the main magnetic field.

**Phase encode gradient**

The concept of phase encoding is somewhat not very easy to understand and requires a little more effort. A proper knowledge of this pays dividend in terms of an overall understanding of the subject.

Assuming we have an MR signal which has all nuclear dipoles in phase. If a phase encoding gradient
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Figure 3.7: Phase encoding returns the signal to Larmor frequency but with position-dependent phase changes.

$G_y$, is applied at a time $t$, in the $y$ direction, then the precession of the nuclear dipoles will speed up or slow down in accordance with their position along the $y$-axis. This causes the nuclear dipoles to dephase to a progressively greater degree depending on the duration of the applied gradient. Fig (3.6) shows the effect of the phase encoding gradient on the transverse magnetisation at three different locations and times [13].

At a different time $t_1$, when the gradient is removed, all the nuclear dipoles will revert to their original frequencies, but will retain their phase angles. They are said to be phase encoded. Relatively, the phase differences between signals at different locations remain until either the application of another gradient or decay of the MR signal due to $T_2$ relaxation [13]. Figure (3.7) shows the phase encoding generated by three different gradient amplitudes on a column of dipoles in the phase-encode axis. We observe that in the absence of an applied gradient, all the nuclear dipoles are in phase and a large signal is obtained, but the dephasing increases with gradient strength until it gets to a large value for all the spins to cancel each other and no spin is obtained.

At different times using a given interval, a second gradient is applied with known tipping angles. This further increases the angle sequel to the initial tipping. More phase encoding steps can be made following the same procedure. The signal amplitudes of each nuclear dipole is resolved along the $y$-axis and the total summed for each column. A set of equation can be derived from this and is solved to get the individual signal amplitudes.

The question now is, How do we measure spatial frequencies from this? For a uniform distribution of nuclear dipoles as assumed, we apply a sufficient phase-encode gradient to cause the phase of the
nuclear dipoles to vary by $3 \times 360^\circ$. When we add up the MR signal from this column we get zero as the dipoles are evenly distributed throughout each direction [13]. It can be said that this object contains no information at the spatial frequency of three cycles per unit length. When we consider a line-pair with alternating sections containing protons or nothing with a spatial frequency of three cycle per unit length. Clearly, only those portions containing nuclear dipoles can contribute to the signal. This implies that a particular value of phase-encode gradient is sensitive to objects containing the spatial frequency three cycles per unit length, but not to others.

Generally, an object will have a range of spatial frequencies. Each value of phase encoding can be considered as a template or a comb that only responds to one spatial distribution of MR signal or spatial frequency. To build up a whole picture, the entire range of possible spatial frequencies has to be interrogated in this way. In the absence of an applied gradient, signal is got from the whole object, and this is referred to as the zero spatial frequency or zero $k$.

For our supposition of a uniform phase for all the nuclear dipoles in the absence of an applied gradient, we shall assign various signal amplitudes, say $a$, $3a$, and $2a$ to individual dipoles along the first column in the phase encode axis to a three step phase encode gradient. As a first phase encode step, we shall tip the proton nuclear magnetic dipoles to a known angle, followed by other steps as shown in figure (3.9).

If we set $a = y_1$, $3a = y_2$, $2a = y_3$, $\theta_{11} = 45^\circ$, $\theta_{12} = 90^\circ$, $\theta_{13} = 135^\circ$, $\theta_{21} = 90^\circ$, $\theta_{22} = 180^\circ$, $\theta_{23} = 270^\circ$, $\theta_{31} = 135^\circ$, $\theta_{32} = 270$, and $\theta_{33} = 405^\circ$; The sum of the individual signal amplitude
These set of equations can be solve completely to calculate $y_1$, $y_2$, and $y_3$. This is true since we know the total signals amplitude from the readout signals i.e $-0.7071a$, $-3a$, and $0.7071a$ (see frequency encoding below), and we also know all the tipping angles for any applied gradient. The MR imaging sequence consists of multiple repetition of the excitation process followed by different phase encoding gradient until possible spatial frequencies are interrogated. Once all the signals are collected and voxels created using the phase and frequency encoding data (raw data in k-space), the computer is used to convert the spatial frequency distribution into a spatial distribution of the excited nuclei i.e an image of the patient.

**Frequency encoding gradient $G_x$ (Readout gradient)**

In frequency encoding, we can acquire all the frequency information needed from one MR signal following one RF pulse excitation, unlike the phase encoding where one MR excitation is required.
for every line of data [13]. Figure (3.10) shows a gradient being applied for a certain time, given a certain gradient moment and a phase change, after which the signal strength is measured. The next data point is measured after a different gradient step. We then obtain data points corresponding to the strength of the MR signal after a whole range of gradient moment.

If we apply the gradient continuously, and measure the MR signal at various time points during the application of that gradient. The MR signal at each point is affected by different amount of gradient moment and yields a different amount of phase change. We observe at each data point a different amount of “phase encoding” and corresponding spatial frequencies.
Spatial encoding

The gradient fields $G_z$, $G_x$ and $G_y$ are applied in a special sequence. As a first step, the slice position and thickness are encoded, then the $x$ and $y$ position in the image matrix follows. The sizes of the image matrix used are typically $128 \times 128$, $256 \times 256$, or $512 \times 512$ voxels.

For the frequency encoded slice axis, the protons exposed to the gradient field will exhibit different Larmor frequencies in accord with their various positions along it. A particular slice can be selected by choosing the precise frequency of RF pulse corresponding to its position described by equation (3.38). Figure (3.6) shows slice selection using a finite bandwidth RF excitation, centered at the Larmor frequency of combined static magnetic field, with the field gradient. The thickness of the slice can be altered by increasing the range of the frequency in the pulse or altering the gradient slope while the bandwidth is constant.

To excite a sharp edge slice, a special pulse shape is used. This is the Sinc-pulse shown in figure (3.10), and is a mathematical function written as $\frac{\sin \pi x}{\pi x}$ [3]. For practical implementation of this pulse, it is necessary to limit the length of the pulse. Thus, five lobes sinc function is used. The precisely shaped sinc-pulse is amplified to give 90 degree or 180 degree power level or even intermediate when smaller tipping angles are used.

After the slice selection, an image matrix can be generated by encoding phases and frequencies along the $y$ and $x$ direction respectively. The $G_y$ gradient is applied or switched on for a short time to enable another frequency change in the signal depending on its position along the vertical $y$-axis. When this is switched off, a return to its former frequency state leads to a phase non-uniformity.
Figure 3.11: A fully encoded matrix where the phase $\phi$, and frequency $\omega$, separate each voxel. This matrix is in k-space.

This dephasing is a result of the shift in their phases according to their position on the y-axis, and is proportional $G_y$. Large phase differences are observed at both ends of this gradient. This field, unlike the $G_x$ and $G_z$ fields, is switched in a series of steps so that the full matrix can be decoded [3]. If there are 128 matrix encoding steps, each step will cause a phase shift of $\frac{360}{128}$ or 2.8 degrees with respect to its neighbor.

The $G_x$ gradient is encoded in a similar manner to the $G_z$ field strengths, but with the $G_x$ corresponding to its position along the x-axis. This is known as the readout gradient, and is allowed on, while the signal frequency measurements are recorded. Having measured the changes in phase and frequency of the NMR signals, further analysis is done using the fast Fourier transform.
Chapter 4

THE FOURIER TRANSFORMS

Fourier transform is a mathematical operation which converts the measured signals, to amplitudes at various frequencies. There is an exact assignment of signals to their corresponding spatial positions. This is because the known gradient provides the relationship between the frequency and position. As a result a projection of an object at an angle is obtained. At any position along the array of the sample, the amplitude is a result of the sum of the amplitude of the nuclear magnetic dipoles within a column of the tissue volume, and depends on spin density, and the $T_1$ and $T_2$ relaxation phenomena.

Following the slice selection, the application frequency-and phase-encoding gradients enable the MR signal to encode spatial frequencies. The effect of the frequency encode gradient $G_x$, applied along the $x$ axis following the excitation of discrete signal element $∂S(t)$ is:

$$∂S(t) = ρ(x).\exp\left(-\frac{t}{T_2}\right).\exp(iγxG_xt)dx$$

(4.1)

$ρ(x)$ is the proton density along $x$. This is the readout gradient. It is applied continuously during the signal acquisition.

The phase encoding gradient $G_y$ is applied along the $y$ axis for a duration of $t_1$. The signal from a small element following the application of both is:

$$∂S(t) = ρ(x, y).\exp(iγxG_xt).\exp(iγyG_yt_1).\exp\left(-\frac{t}{T_2}\right)dxdy$$

(4.2)

This implies that, signal=spin density $× T_2$ decay $×$ phase change due to $G_x$ $×$ phase change due to $G_y$. The total MR signal is the integral of this with respect to $x$ and $y$. 

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Two-Dimensional Fourier spin-echo imaging

The sequences of a two-dimensional spin-echo pulse can be summarised thus:
A 90 degree RF pulse is tuned to a narrow frequency bandwidth in conjunction with the slice selection gradient. This converts longitudinal magnetisation to transverse magnetisation, and produces the greatest signal than all other tipping angles. Subsequently, a phase difference among the nuclear magnetic dipoles is induced by a phase encoding gradient in accordance with their positions in the phase encode direction. After a delay time, $T_E$, the 180 degree RF pulse is applied to rephase this dephasing, and an echo is formed at an echo time, $T_E$. Just before and during the echo signal detection, the frequency encode gradient is applied orthogonally to the slice select and phase encode directions. This causes a dispersion of the precessional frequency of the nuclear dipoles, and the frequencies depend on their positions along the readout gradient. At the same time, the signals are converted to digital numbers and stored in k-space two-dimensional array in a computer memory.

The k-space matrix is a frequency domain description of the image data, with origin at the center, negative frequencies to the left and below and positive to the right and above. The proton positions are encoded by variations in frequency (x-axis) and phase (y-axis). The spatial domain image, is the decoding of the frequency domain by the Two-dimensional Fourier transform, which displays spatial anatomic relations. Finally, the image is a spatial representation of $T_1$, $T_2$ and spin density characteristics of the proton magnetisation of tissues.

Derivation of image equation

We shall consider the general case of spin-density distribution $\rho(x, y, z)$ measured in an MRI system alongside the three orthogonal gradients $G_x$, $G_y$ and $G_z$. After the application of the 90 degree pulse, the signal at a time $t$ is $S(t)$, and is written as:

$$S(t) = CM_0 \int_x \int_y \int_z \rho(x, y, z) \exp(i\gamma \int_0^t [xG_x(t') + yG_y(t') + zG_z(t')] dt) \exp(-\frac{t}{T_2}) dxdydz$$

(4.3)

Where $\omega_x = \gamma xG_x$; $\omega_y = \gamma yG_y$ and $\omega_z = \gamma zG_z$ and $C$ is a constant which is a function of the geometry of the coil. Thus,

$$S(t) = CM_0 \int_x \int_y \int_z \rho(x, y, z) \exp[i(\omega_x + \omega_y + \omega_z)t] \exp(-\frac{t}{T_2}) dxdydz$$

(4.4)

The form shown above is a three-dimensional Fourier transform. Thus, the spin distribution can be got using inverse three dimensional transform. Most MRI imaging simplifies the read-out of the signal by reducing the number of gradients. In most cases, a transaxial slice is being reconstructed, and so $\omega_z$ will be zero as $z$ selection excitation would have been applied to a sample for the slice to be imaged. Therefore, this reduces the gradients to a combination of $x$ and $y$, thus:

$$S(t) = CM_0 \int_x \int_y \int_z \rho(x, y) \exp[i\gamma(xG_x + yG_y)t] \exp(-\frac{t}{T_2}) dxdy$$

(4.5)
In order to perform a two-dimensional Fourier transform reconstruction following the slice selection in the $z$ plane, a constant gradient is applied in the $x$ direction for the time $t_1$, for the frequency encoding of the readout signal. The amplitude of $G_y$ with an exposure time $t_2$ will vary for different cycle of the pulse sequence. Thus:

$$S(t_1, t_2) = C M_0 \int_x \int_y \rho(x, y) \exp[i\gamma(xG_x t_1 + yG_y t_2)] \exp(-\frac{t}{T_2^*})dxdy \quad (4.6)$$

An application of a two-dimensional transform results in the spin distribution $\rho(x, y)$ in the selected slice along the $z$-direction. The figure (4.1) shows the raw data set obtained with the gradient echo sequence and the image obtained following the Fourier transformation.
Conclusion

We have discussed the basic principles of MRI, beginning with an understanding of the constituents of the nucleus of an atom, and how the manifestation of a quantum property, “spin”, is dependent on their numbers. The spins create nuclear magnetic dipoles which are aligned at discrete angles relative to the external static magnetic field. The alignment of these nuclear dipoles in the external field direction yields a net magnetisation vector, which is along the Z-direction (external magnetic field direction). This is known as longitudinal magnetisation.

If a second magnetic field with a lower magnetic field strength is applied in the plane perpendicular to the axis of the static magnetic field, there is an increase in the spin state of a nuclear dipole which absorbs such a photon, and the angle of the dipole axis with respect to the static field increases by a discrete amount. As the duration of exposure with this second magnetic field increases, there is an increase in the number of the nuclear dipoles that attain greater spin state. The magnetisation vector tips with respect to the static external magnetic field. When the second magnetic field is removed, the overall distribution of spin states of the nuclear dipole begins to return to its original state. The net magnetisation tends towards realigning itself with the first external magnetic field. Due to this relaxation, the rates at which the statically aligned vector increases and the transverse magnetisation vector decreases, can be measured. These rates are characterised by the $T_1$ and $T_2$ relaxation times respectively.

The frequencies and phases are forced by the magnetic field gradients to be spatially dependent. Raw data are obtained in k-space and converted by Fourier transform to a pictorial representation.
References


